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A Comparison Of Strong And Weak Coupling Schemes For Computational Aeroelasticity In OpenFOAM

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Motivation

- Developed an implicit density-based fluid solver for high fidelity transonic aeroelastic analysis
- Based on existing explicit solver in OpenFOAM, an open source C++ based toolbox primarily for CFD
- Poor convergence when coupled with the OpenFOAM 6DoF rigid body structural solver

- **Goal:** Construct improved coupling schemes for rigid body fluid-structure interaction

Outline

1) Fluid Solver – Implicit *rhoCentralFoam*
2) Structural Solver – *sixDoFRigidBodyMotion*
3) Coupling Schemes
4) Test Cases and Convergence
5) Stability and Accuracy
6) Computational Cost

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Fluid Solver – *rhoCentralFoam*

- Euler equations: $$\frac{\partial}{\partial t}\int_{\Omega}\mathbf{W}d\Omega + \oint_{\partial\Omega}\mathbf{F}_c dS = 0$$

$$\mathbf{W} = \begin{pmatrix} \rho \\ \rho\mathbf{U} \\ \rho E \end{pmatrix} \qquad \mathbf{F}_c = \begin{pmatrix} \rho\mathbf{U}\cdot\mathbf{n} \\ \rho\mathbf{U}(\mathbf{U}\cdot\mathbf{n}) + p\mathbf{n} \\ (\rho E + p)\mathbf{U}\cdot\mathbf{n} \end{pmatrix}$$

Conservative variables — Convective flux

- Discretization: $$V_i\left(\frac{3\mathbf{W}_i^{n+1} - 4\mathbf{W}_i^{n} + \mathbf{W}_i^{n-1}}{2\Delta t}\right) + \sum_{faces(i)}\mathbf{F}_c S_f = 0$$

- Central-upwind scheme of Kurganov, Noelle and Petrova

Enhancements to *rhoCentralFoam*

1) Implicit formulation using analytically derived flux Jacobian

$$\left[\mathbf{F}_c^M\right]^{n+1} = \left[\mathbf{F}_c^M\right]^n + \left[\frac{\partial \mathbf{F}_c^M}{\partial \mathbf{W}}\right]^n (\mathbf{W}^{n+1} - \mathbf{W}^n) + O\{(\Delta \mathbf{W})^2\}$$

2) Dual time stepping
   - Inner iterations to ensure unsteady convergence

3) Discretized equations solved in a coupled manner using Lower-Upper Symmetric Gauss-Seidel (LU-SGS)

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Structural Solver - *sixDoFRigidBodyMotion*

$$\begin{bmatrix} \mathbf{M} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{J_{ea}} \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{h}} \\ \ddot{\boldsymbol{\alpha}} \end{pmatrix} + \begin{bmatrix} \mathbf{D}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_\alpha \end{bmatrix} \begin{pmatrix} \dot{\mathbf{h}} \\ \dot{\boldsymbol{\alpha}} \end{pmatrix} + \begin{bmatrix} \mathbf{K}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_\alpha \end{bmatrix} \begin{pmatrix} \mathbf{h} \\ \boldsymbol{\alpha} \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ \boldsymbol{\tau}_{ea} \end{pmatrix}$$

- Leapfrog integration of rigid body dynamics
  - Explicit, second order, symplectic (energy preserving)

1) First half step $\dot{\mathbf{h}}^{n+\frac{1}{2}} = \dot{\mathbf{h}}^{n} + \frac{1}{2}\Delta t\ \ddot{\mathbf{h}}^{n}$
2) Update position $\mathbf{h}^{n+1} = \mathbf{h}^{n} + \Delta t\ \dot{\mathbf{h}}^{n+\frac{1}{2}}$
3) Update acceleration $\mathbf{compute}\ \ \ddot{\mathbf{h}}^{n+1}$
4) Second half step $\dot{\mathbf{h}}^{n+1} = \dot{\mathbf{h}}^{n+\frac{1}{2}} + \frac{1}{2}\Delta t\ \ddot{\mathbf{h}}^{n+1}$

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Baseline Weak Coupling

1) First half step $\dot{\mathbf{h}}^{n+\frac{1}{2}} = \dot{\mathbf{h}}^{n} + \frac{1}{2}\Delta t\ \ddot{\mathbf{h}}^{n}$
2) Update position $\mathbf{h}^{n+1} = \mathbf{h}^{n} + \Delta t\ \dot{\mathbf{h}}^{n+\frac{1}{2}}$
3) Update acceleration $\mathbf{compute}\ \ \ddot{\mathbf{h}}^{n+1}$
   a) new spring and damping forces (time n+1)
   b) <u>old</u> fluid forces (time n)
4) Second half step $\dot{\mathbf{h}}^{n+1} = \dot{\mathbf{h}}^{n+\frac{1}{2}} + \frac{1}{2}\Delta t\ \ddot{\mathbf{h}}^{n+1}$

5) Fluid solve

Modified Weak Coupling

1) First half step $\dot{\mathbf{h}}^{n+\frac{1}{2}} = \dot{\mathbf{h}}^{n} + \frac{1}{2}\Delta t\ \ddot{\mathbf{h}}^{n}$
2) Update position $\mathbf{h}^{n+1} = \mathbf{h}^{n} + \Delta t\ \dot{\mathbf{h}}^{n+\frac{1}{2}}$
3) **Fluid solve**
4) Update acceleration $\mathbf{compute}\ \ \ddot{\mathbf{h}}^{n+1}$
   a) new spring and damping forces (time n+1)
   b) **<u>new</u>** fluid forces (time n+1)
5) Second half step $\dot{\mathbf{h}}^{n+1} = \dot{\mathbf{h}}^{n+\frac{1}{2}} + \frac{1}{2}\Delta t\ \ddot{\mathbf{h}}^{n+1}$

Strong Coupling

- Crank-Nicolson (or average acceleration method)
  - Implicit, second order, symplectic (energy preserving)

1) **Fluid solver dual time iteration**

2) Estimate acceleration $\quad \mathbf{compute} \quad \ddot{\mathbf{h}}^{n+1}$

3) Estimate velocity $\quad \dot{\mathbf{h}}^{n+1} = \dot{\mathbf{h}}^{n} + \frac{1}{2}\Delta t \left(\ddot{\mathbf{h}}^{n+1} + \ddot{\mathbf{h}}^{n} \right)$

4) Estimate position $\quad \mathbf{h}^{n+1} = \mathbf{h}^{n} + \frac{1}{2}\Delta t \left(\dot{\mathbf{h}}^{n+1} + \dot{\mathbf{h}}^{n} \right)$

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Test Cases

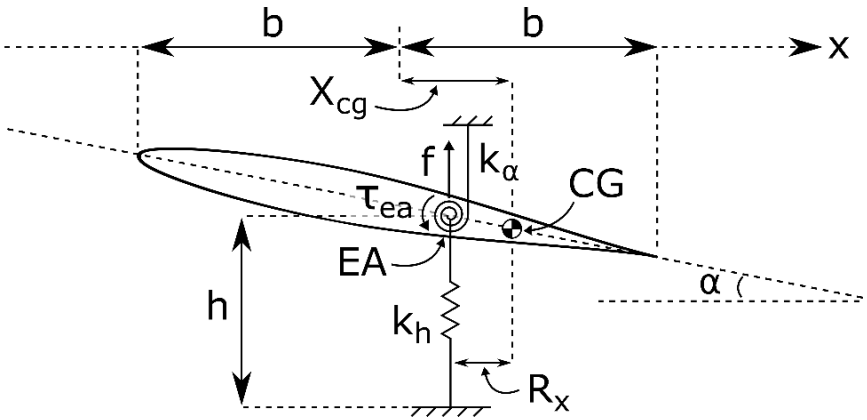
- Two flutter cases
  - 2D Elastic NACA 64A010
  - 3D Benchmark Supercritical Wing (BSCW)

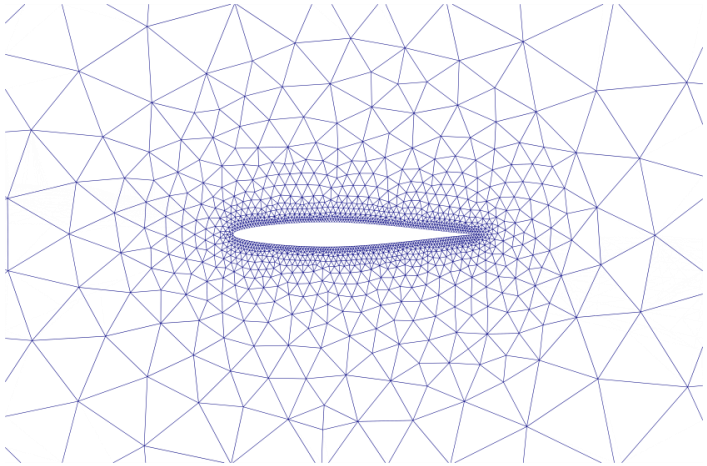
- Evaluate sensitivity of aeroelastic solutions to changes in time step size
- Solutions characterized by damping ratio of dominant aeroelastic mode

Elastic NACA 64A010

- Aeroelastic model introduced by Isogai (1979, 1981)
- 2DoF model of an outboard section of sweptback wing

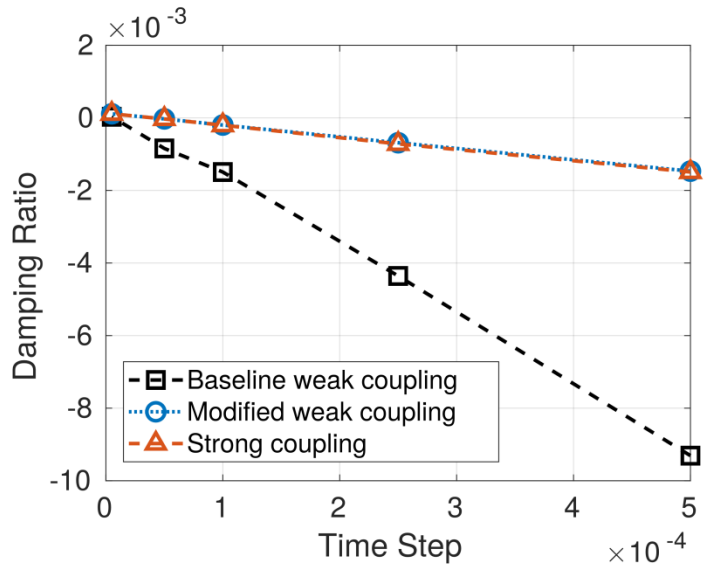
- M = 0.825
- Speed index (V_f) = 0.641

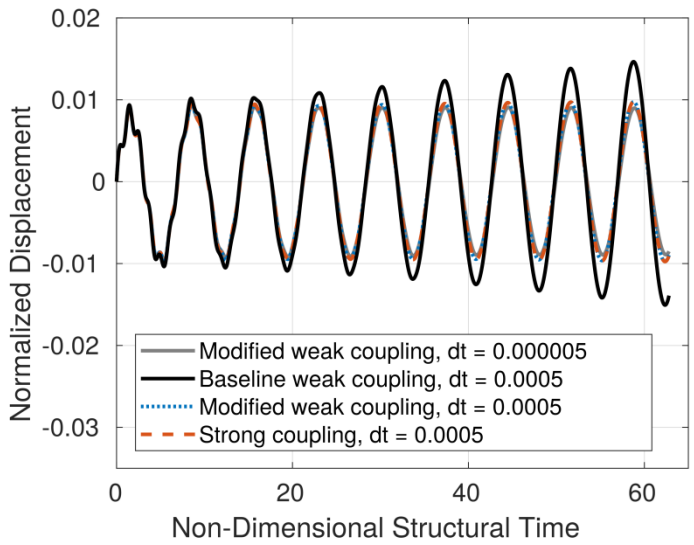




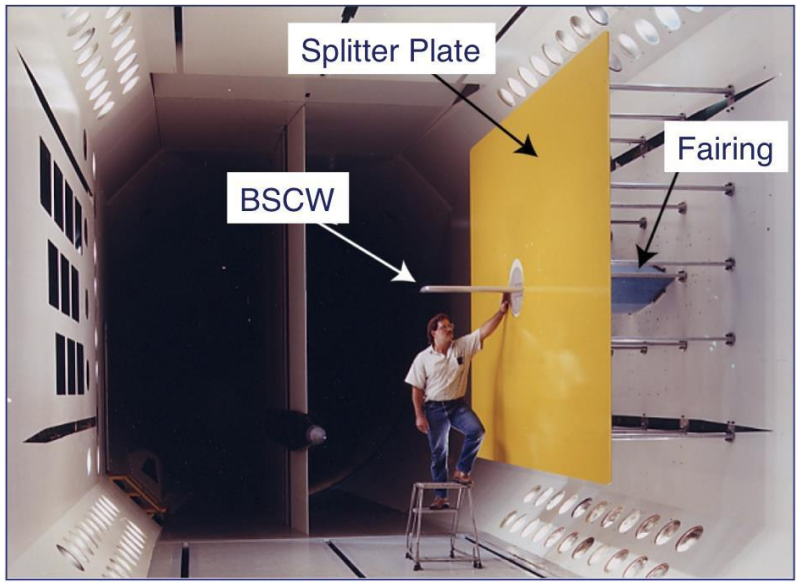
Elastic NACA 64A010 Convergence

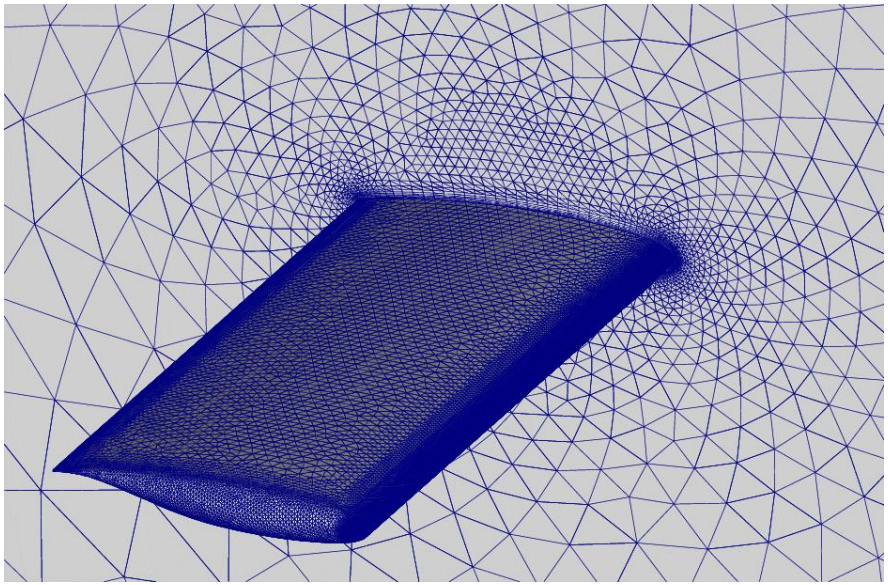
- Two newly implemented schemes produce similar results and significantly outperform the baseline scheme





Benchmark Supercritical Wing (BSCW)





- Rigid rectangular wing
- NASA SC(2)-0414 airfoil section

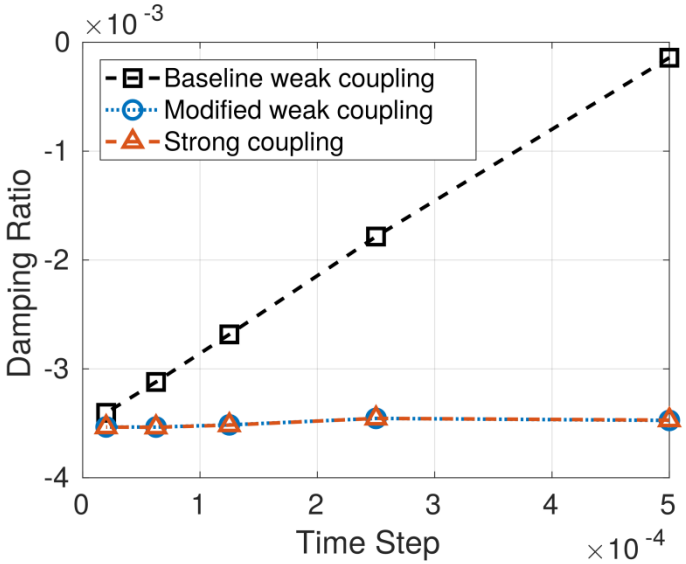
BSCW Flutter Case from AePW-2

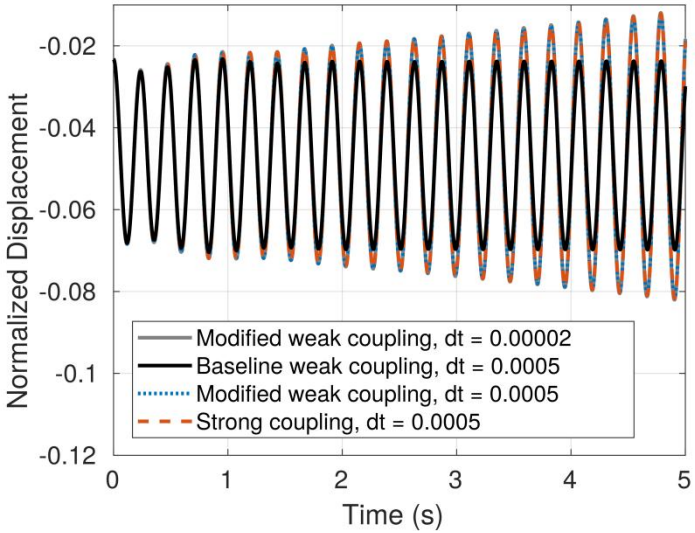
	Case #1	Case #2	Optional Case #3a	Optional Case #3b	Optional Case #3c
Mach	0.7	0.74	0.85	0.85	0.85
AoA	3°	0°	5°	5°	5°
Dynamic Data Type	Forced Oscillation f = 10Hz, $\|\theta\|$=1°	Flutter	Unforced Unsteady	Forced Oscillation f = 10Hz, $\|\theta\|$=1°	Flutter
Notes:	- Attached flow - OTT exp. data - R-134a	- Flow state(?) - PAPA exp. data - R-12	- Separated flow - OTT exp. data - R-134a	- Separated flow - OTT exp. data - R-134a	- Separated flow - No exp. data - R-134a

- Pitch and heave degrees of freedom
- P_d = 166 psf

BSCW Convergence

- Results similar to NACA 64A010 case





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Stability and Accuracy

	Symplectic	Non-symplectic
Minimal Time Lag	Modified Leapfrog Crank-Nicolson	
One Time Step Lag	Baseline Leapfrog	

Stability and Accuracy

	Symplectic	Non-symplectic
Minimal Time Lag	Modified Leapfrog Crank-Nicolson	
One Time Step Lag	Baseline Leapfrog	AB2T

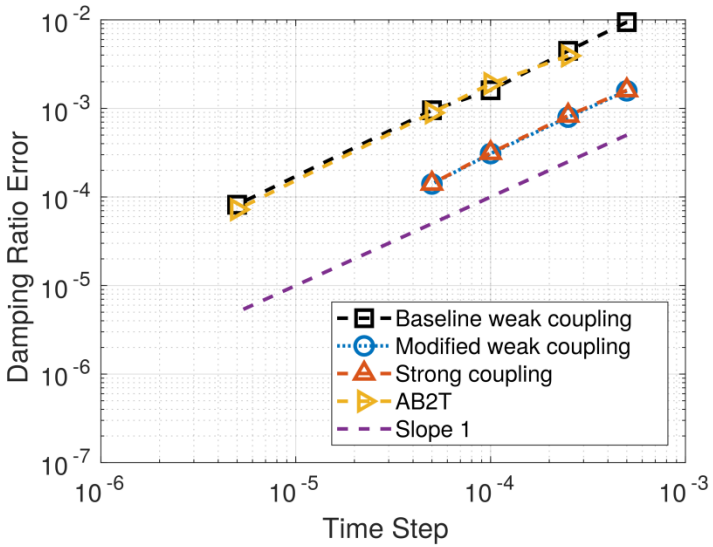
- AB2T – simplistic weak coupling scheme
- 2-step Adams-Bashforth for velocity
- Trapezoid rule for position

$$\dot{\mathbf{h}}^{n+1} = \dot{\mathbf{h}}^{n} + \tfrac{1}{2}\Delta t\left(3\ddot{\mathbf{h}}^{n} - \ddot{\mathbf{h}}^{n-1}\right)$$

$$\mathbf{h}^{n+1} = \mathbf{h}^{n} + \tfrac{1}{2}\Delta t\left(\dot{\mathbf{h}}^{n+1} + \dot{\mathbf{h}}^{n}\right)$$

Stability and Accuracy

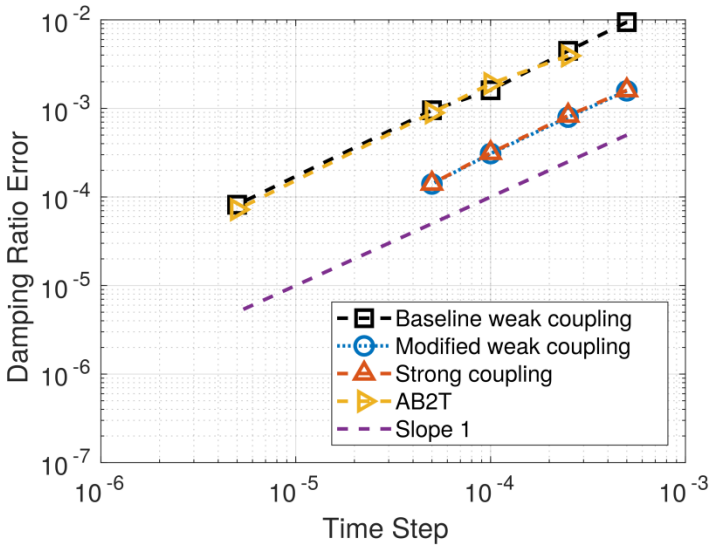
- Damping ratio error convergence for NACA 64A010

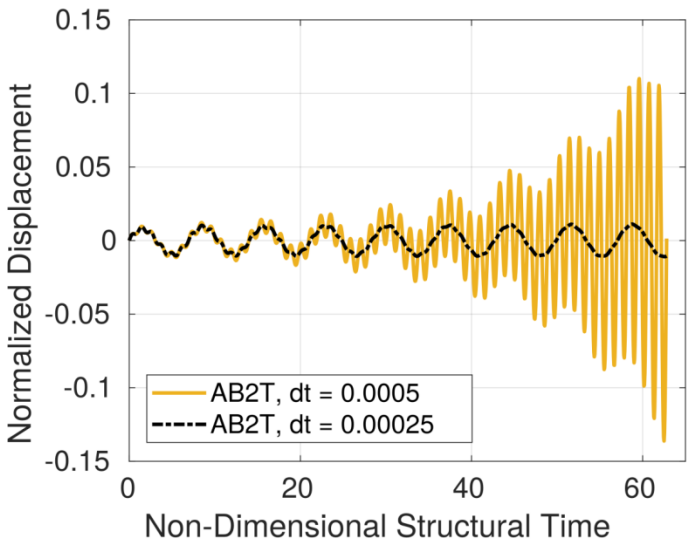


- Minimizing time lag → higher accuracy

Stability and Accuracy

- Damping ratio error convergence for NACA 64A010

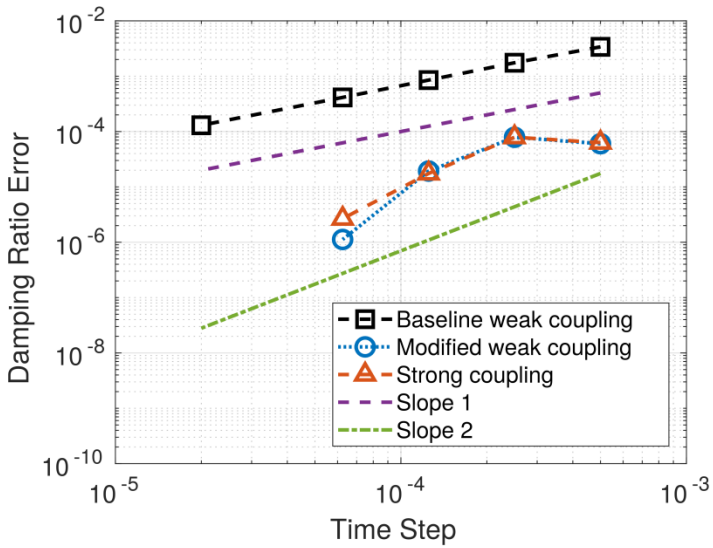




- Minimizing time lag → higher accuracy
- Symplectic → stable for large time steps

Stability and Accuracy

- Damping ratio error convergence for BSCW



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Computational Cost

- Strong coupling involves solving structural equations at every subiteration
- Sizeable computational cost associated with integrating pressure forces and updating mesh points for large meshes
- E.g. One case with strong coupling took 20% longer than with modified weak coupling for BSCW

Summary

- Formulated two coupling schemes for rigid body fluid-structure interaction in OpenFOAM
  - Leapfrog weak coupling modified to minimize time lag between fluid and structural solvers
  - Crank-Nicolson strong coupling using dual time iterations
- Two improved schemes show much improved convergence for aeroelastic simulations of NACA 64A010 and BSCW compared to the baseline
- Leapfrog weak coupling can be as accurate as Crank-Nicolson strong coupling with lower computational cost