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# *Advanced Sonic Boom Analysis Using the Full-Field Simulation*

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# 1. Background

## Sonic Boom

Acoustic phenomenon by shocks

⇒ Explosive noise on the ground

⇓

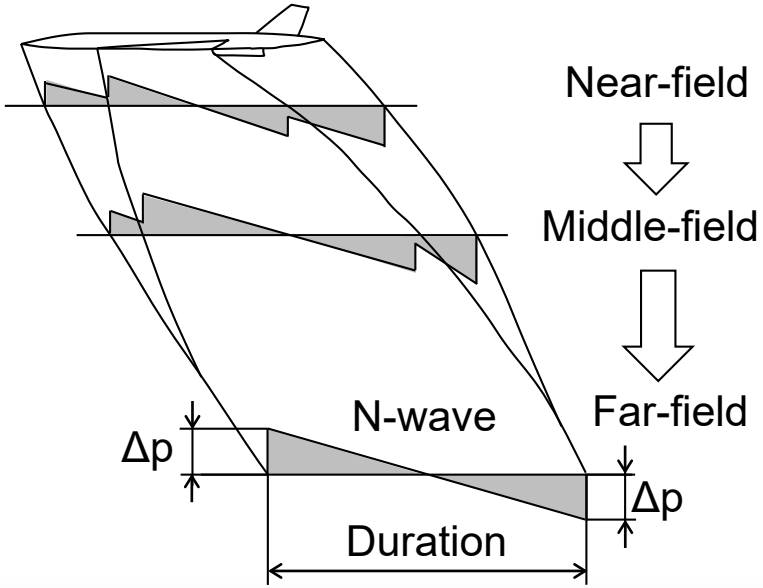
Sonic Boom reduction is essential

## Sonic Boom Intensity

Depend on many factors

- Aircraft configuration
- Flight and atmos. conditions
- Ground topography

http://www.aero.jaxa.jp/research/frontier/sst/



# 1. Background

## Sonic boom prediction methods

- Waveform Parameter Method

  Representative method based on the geometric acoustics

- Latest prediction methods in various conditions
  - Augmented burgers eq.

    Sonic boom prediction during steady flight
  - Lossy nonlinear Tricomi eq.

    Focused boom prediction during acceleration or maneuver
  - HOWARD approach

    Evaluation of atmospheric turbulence effect

The methods above are useful for predicting sonic boom strength, but some sonic boom phenomena would be difficult to evaluate

# 1. Background

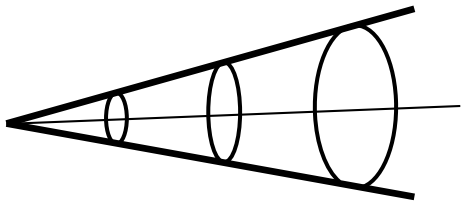
## Full-Field Simulation

CFD analysis in entire flow fields, including the near field, far field, and caustic-vicinity field

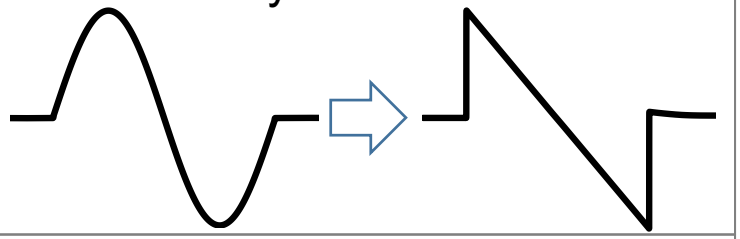
## To realize full-field simulation

Capturing shock waves in the entire domain, reducing comp. load, and considering the following effects

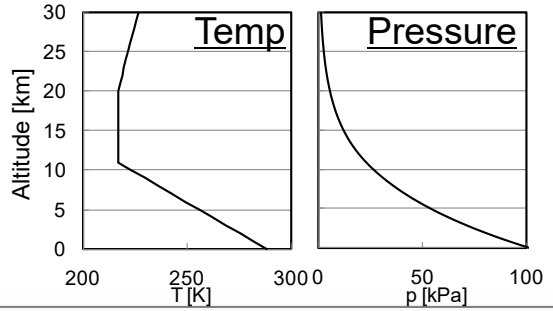
- Geometrical spreading



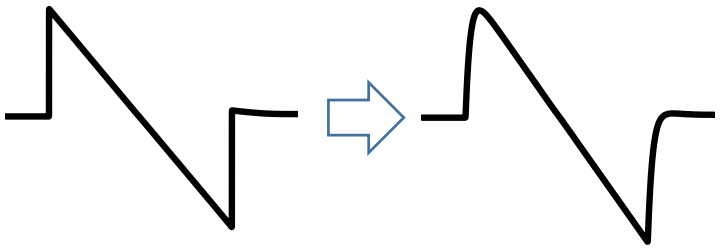
- Nonlinearity



- Atmospheric stratification




- Molecular relaxation, viscosity

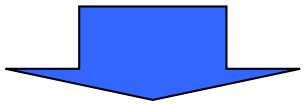


# 1. Background

## Objectives

To formulate framework of full-field simulation method, and to clarify complex sonic boom phenomena as follows:

- Cutoff phenomena at low supersonic speed
- Waveform transition at hypersonic speed



Consequently, full-field simulation method becomes a powerful tool for realizing numerical flight experiment

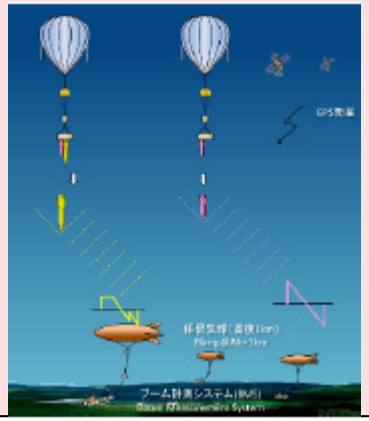
# Roadmap

**Formulation of simulation method**

- ☐ Computational method considering following five effects:
  - ☐ geometrical spreading
  - ☐ nonlinearity
  - ☐ molecular relaxation
  - ☐ atmos. stratification
  - ☐ viscosity
- ☐ Solution-adapted grid generation method
- ☐ Segmentation method of computational domain

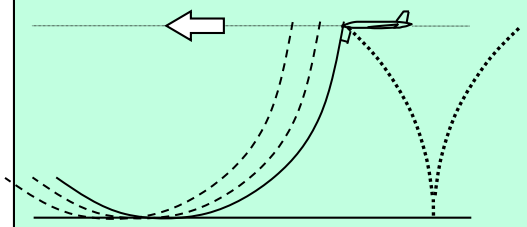
**Validation of full-field simulation method**

- ☐ Reproduction of flight test for sonic boom at supersonic speed
- ☐ Comparison with waveform parameter method

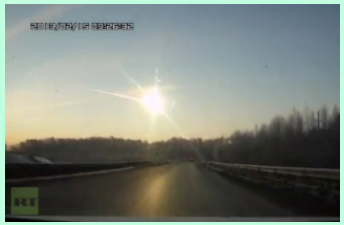


**Demonstration of applicability**

- ☐ Sonic boom cutoff phenomena at low supersonic speed



- ☐ Sonic boom propagation at hypersonic speed



**Requirement for simulation method**

- Considering important physical effects
- Capturing shock waves in entire domain
- Reducing computational load

# 2. Simulation method

## Governing equations

$$\frac{\partial Q}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\frac{\partial G}{\partial z}=\frac{\partial E_v}{\partial x}+\frac{\partial F_v}{\partial y}+\frac{\partial G_v}{\partial z}+S_G+S_W$$

・Geometrical spreading can be considered by 3D analysis

Convective terms (nonlinearity)

Viscous terms (viscosity)

Gravity term (atmos. stratification)

Translational-vibrational relaxation term (Molecular relaxation of $O_2$, $N_2$)

$$Q=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho w\\ e\\ e_{vO_2}\\ e_{vN_2}\end{bmatrix},E=\begin{bmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ \rho uw\\ (e+p)u\\ e_{vO_2}u\\ e_{vN_2}u\end{bmatrix},F=\begin{bmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ \rho vw\\ (e+p)v\\ e_{vO_2}v\\ e_{vN_2}v\end{bmatrix},G=\begin{bmatrix}\rho w\\ \rho uw\\ \rho vw\\ \rho w^2+p\\ (e+p)w\\ e_{vO_2}w\\ e_{vN_2}w\end{bmatrix}$$

$$E_v=\begin{bmatrix}0\\ \tau_{xx}\\ \tau_{xy}\\ \tau_{xz}\\ \beta_x\\ 0\\ 0\end{bmatrix},F_v=\begin{bmatrix}0\\ \tau_{yx}\\ \tau_{yy}\\ \tau_{yz}\\ \beta_y\\ 0\\ 0\end{bmatrix},G_v=\begin{bmatrix}0\\ \tau_{zx}\\ \tau_{zy}\\ \tau_{zz}\\ \beta_z\\ 0\\ 0\end{bmatrix}S_G=\begin{bmatrix}0\\ 0\\ \rho g\\ 0\\ \rho gv\\ 0\\ 0\end{bmatrix},S_W=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ w_{vO_2}\\ w_{vN_2}\end{bmatrix}$$

$$e=\frac{\rho RT}{\gamma-1}+e_{vO_2}+e_{vN_2}+\frac{\rho}{2}\left(u^2+v^2+w^2\right)$$

Translational and rotational temp : $T$

Vibrational temp : $Tv_{O2}$, $Tv_{N2}$

$$e_{vs}(T)=\frac{\varepsilon_s R\theta_s}{\exp(\theta_s/T)-1}$$ (harmonic oscillation model)

$s = O_2, N_2$, $\theta_{O2} = 2273.0$ K, $\theta_{N2} = 3393.0$ K,
$\varepsilon_{O2} = 0.233$, $\varepsilon_{O2} = 0.767$

## Relaxation model

- Landau-Teller rate model : $w_{vs} = \dfrac{e_{vs}(T) - e_{vs}(T_{vs})}{\tau_s}$

- Relaxation time by Bass et al.

$$\frac{1}{\tau_{O_2}} = \frac{2\pi p}{p_0}\left(24 + 4.04\times10^4 h_a \frac{0.02 + h_a}{0.391 + h_a}\right)$$

$$\frac{1}{\tau_{N_2}} = \frac{2\pi p}{p_0}\left\{\left(\frac{T_0}{T}\right)^{\frac{1}{2}}\left[9 + 280 h_a \exp\left(-4.17\left\{\left(\frac{T_0}{T}\right)^{\frac{1}{3}} - 1\right\}\right)\right]\right\}$$

Parameter

$p$ : Pressure
$T$ : Temperature
$h_a$ : Absolute humidity[%]

Standard state : $T_0 = 293.15$ K
$p_0 = 1.013\times10^5$ Pa

Bass, H. E. et al. : Atmospheric absorption of sound : Further developments, J. Acoust. Soc. Am., 97 (1995), pp. 680-683

- Relation of relative and absolute humidity

$$h_a = h_r \frac{p_{sat}/p_0}{p/p_0} = p_0\left(\frac{h_r}{p}\right)\left(\frac{p_{sat}}{p_0}\right)$$

$$\log_{10}\left(\frac{p_{sat}}{p_0}\right) = -6.8346\left(\frac{T_{01}}{T}\right)^{1.261} + 4.6151$$

Paremeter

$h_r$ : Relative humidity[%]

$p_{sat}$ : Saturated vapor pressure
$T_{01} = 273.16$ K (Triple-point)

## Computational approach

- Conventional approach used in compressible CFD analysis is applied

- Convective terms：SHUS＋3rd-order MUSCL
- Viscous terms：2nd-order central difference
- Relaxation term：Diagonal implicit method
- Time integration ： MFGS

- There is no precedent for CFD considering atmospheric stratification

⇒ New computational approach is constructed

- Hydrostatic eq. ：

$$\frac{dp_a}{dh} = -g\rho_a \quad \xrightarrow{\text{Discretization}} \quad \frac{\Delta p_a}{\Delta h} = -g\rho_a + O(\Delta h^2)$$

Discretization error
⇒nonphysical waves generate

- Governing eq. ：

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = \frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} + \frac{\partial G_v}{\partial z} + S_G + S_W$$

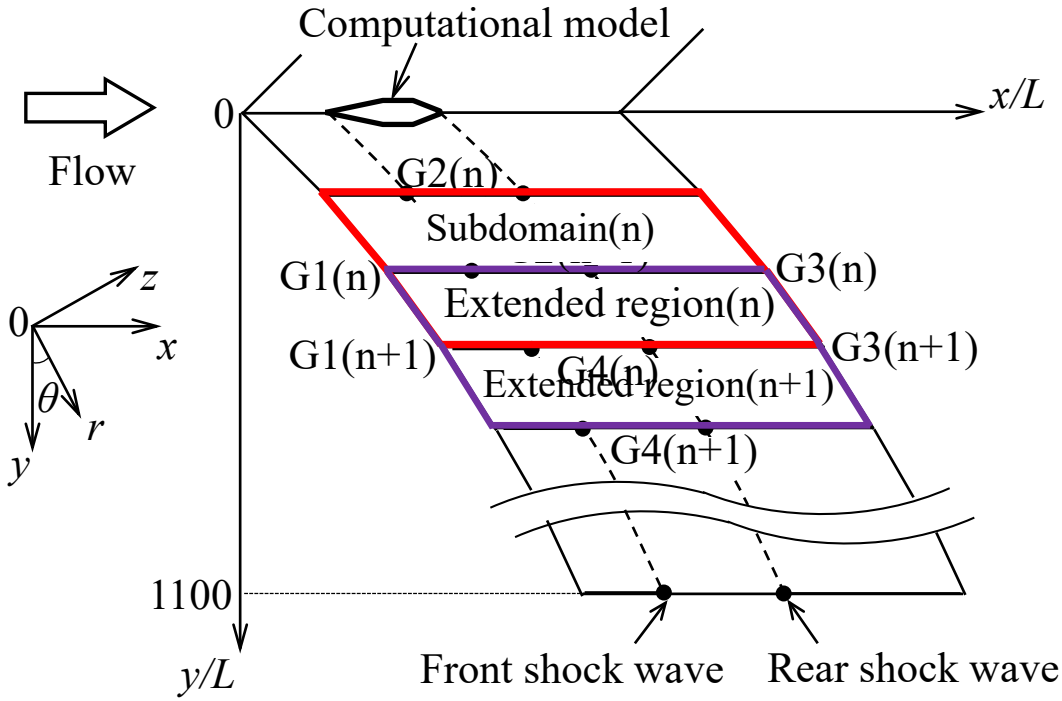
$$\xrightarrow{\text{Discretization}} \quad \left(\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z}\right)_{i,j,k} = \left(\frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} + \frac{\partial G_v}{\partial z} + S_G + S_W + S_C\right)_{i,j,k}$$

Source term　Correction term

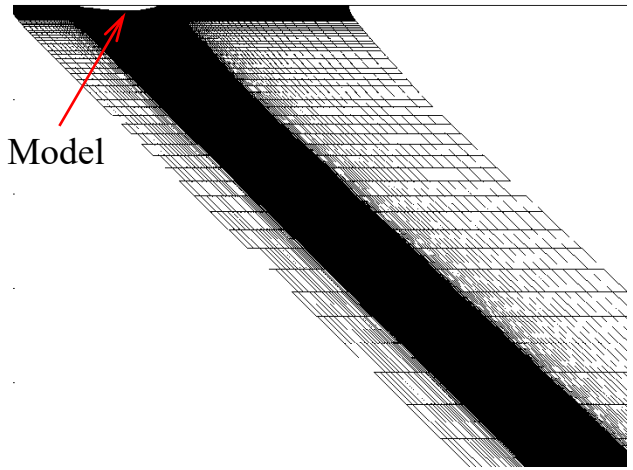
- Yamashita, R. et al, Transactions of JSASS, Vol.58, No.6, pp. 327-336, 2015.

# 2. Simulation method

## Computational grid




Axi-symmetric grid in near field

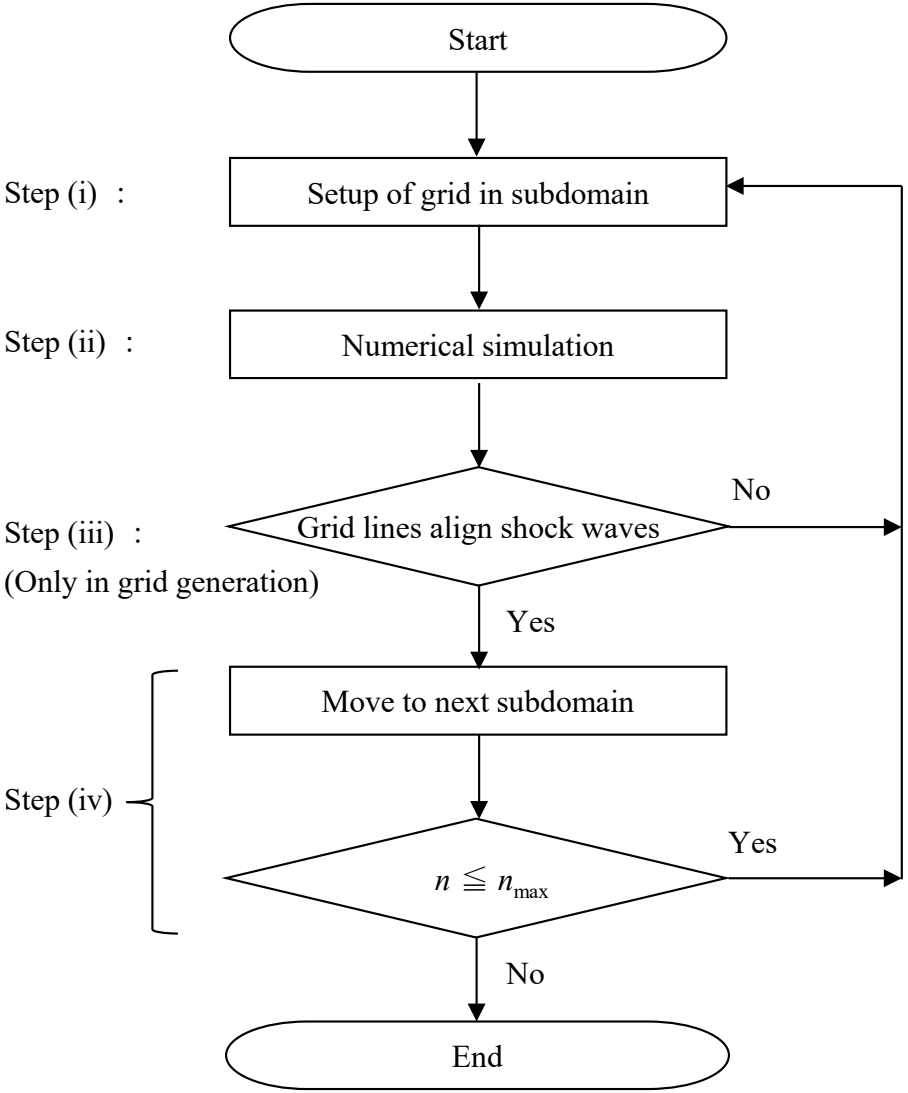


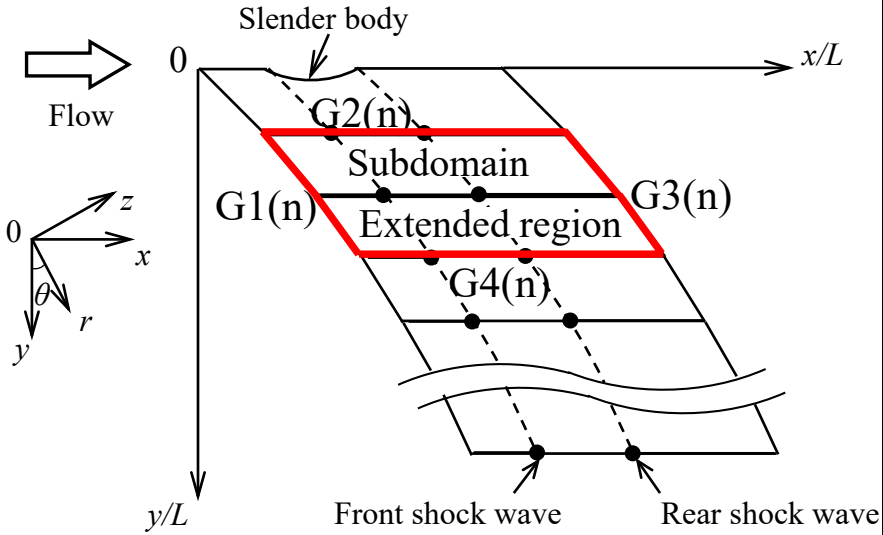

- Boundary condition in r direction
  Special treatment is necessary
  ⇒ r/L = 0-1 ： uniform atmosphere

- 3D adapted grid：constructed rotating 2D grid around body axis
- Entire domain：Subdomain(n)+Extended region(n) （n = 1,2,3,…）
  (Extended region is solved to avoid influence of G4)

⇒ Efficiency of computation is significantly improved

# 2. Simulation method

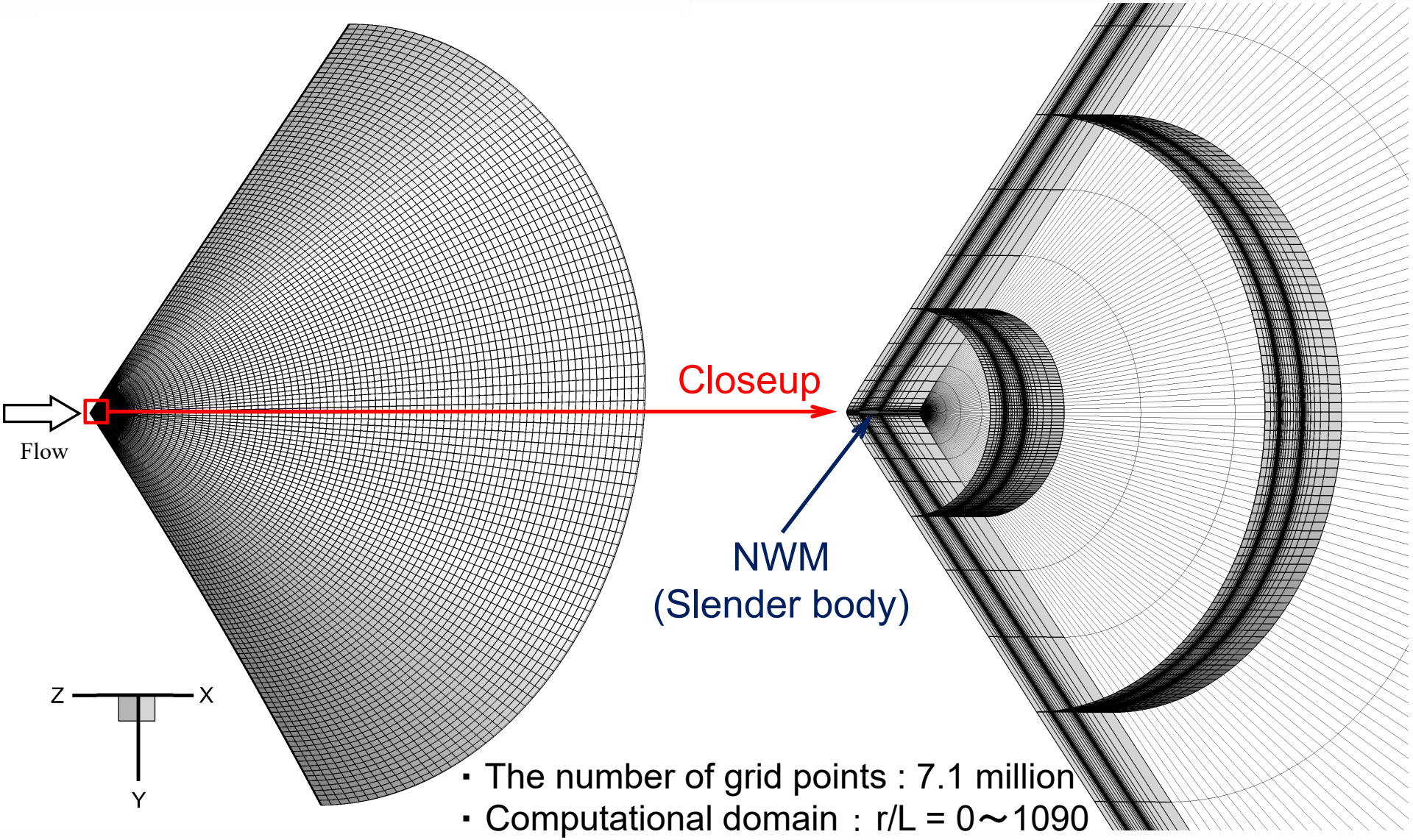



## Computational procedure

① Setup of computational grid
② Numerical simulation
③ Validation of computational grid
（If grid lines do not align shock waves, grid is reconstructed）
④ Move to next subdomain

# 2. Simulation method




- The number of grid points : 7.1 million
- Computational domain : r/L = 0～1090

# Roadmap

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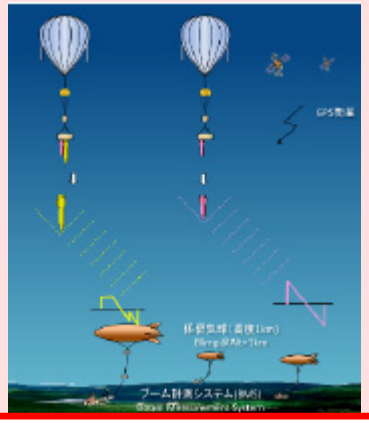
  - molecular relaxation
  - atmos. stratification
  - viscosity
- Solution-adapted grid generation method
- Segmentation method of computational domain

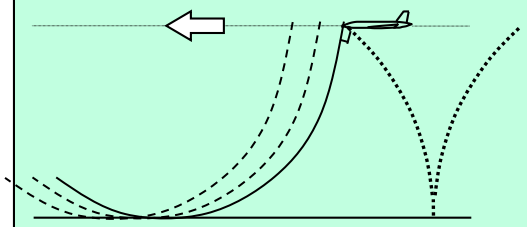
**Validation of full-field simulation method**

- Reproduction of flight test for sonic boom at supersonic speed
- Comparison with waveform parameter method

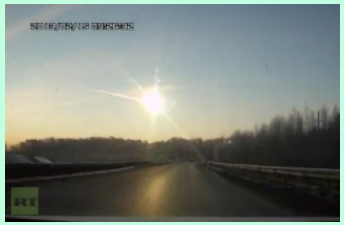


**Demonstration of applicability**

- Sonic boom cutoff phenomena at low supersonic speed



- Sonic boom propagation at hypersonic speed

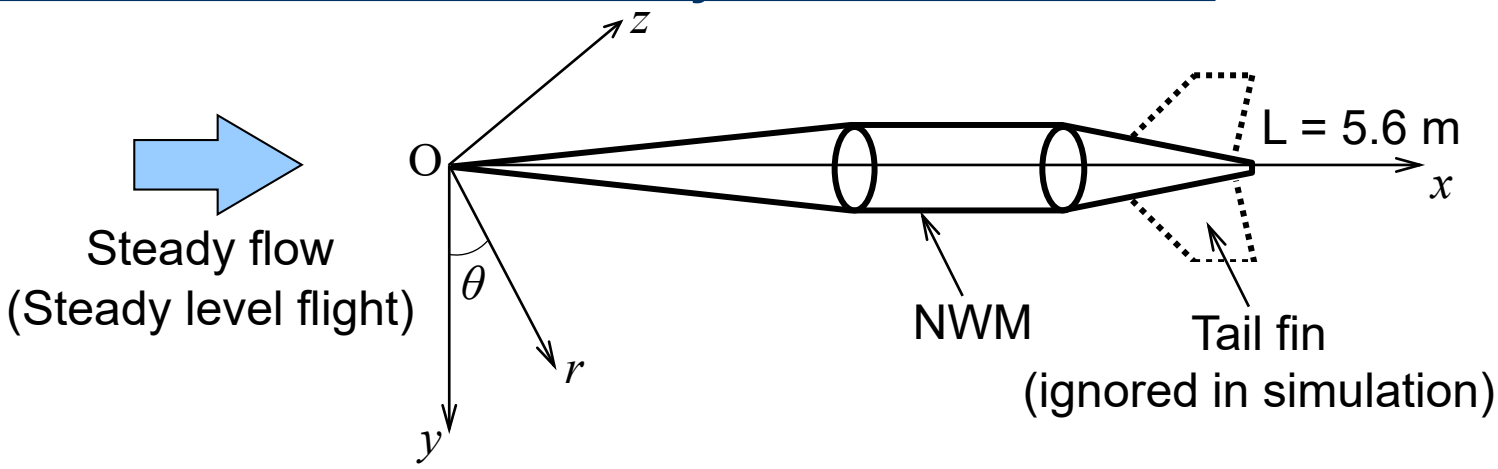


Reproduction of D-SEND#1 flight test by JAXA


- Yamashita R. et al., AIAA Journal, Vol. 54, No. 10, pp. 3223-3231, 2016.
- Yamashita R. et al., Journal of Aircraft, Vol. 55, No. 3, pp. 1305-1310, 2018.

# 3. Simulation accuracy

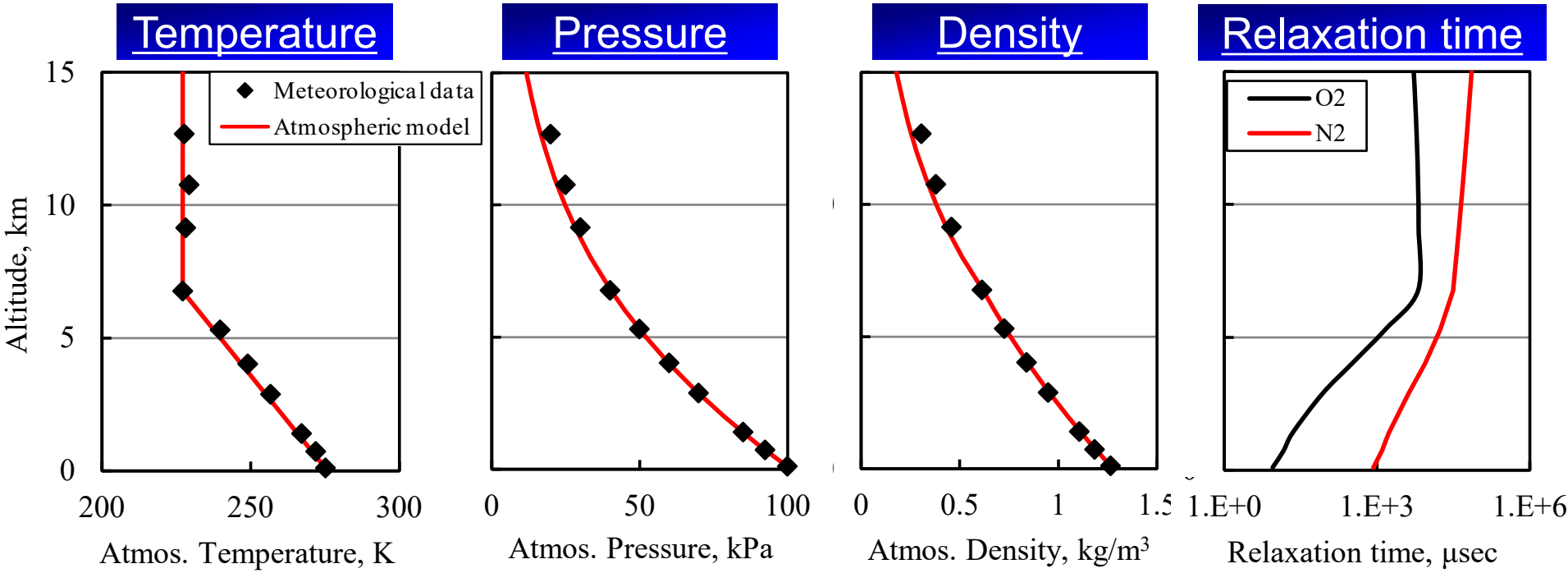
## NWM（N-Wave Model）by JAXA in D-SEND#1




## Numerical condition

| | |
|---|---|
| Flight Mach number | $M_\infty$ = 1.43 |
| Flight altitude | h = 6.039 km |
| Computational domain | r/L = 0～1090<br>($r_{max}$ = 6.104 km) |
| Atmospheric condition | Atmospheric model<br>based on meteorological data |

# 3. Simulation accuracy




- Temperature : $T_a = T_0 - \beta h \quad (h \le 6.75km)$

  : $T_a = const \quad (h \ge 6.75km)$

- Hydrostatic eq. : $\dfrac{dp_a}{dh} = -g\rho_a$

- Eq. of state of ideal gas : $p_a = \rho_a R T_a$

- Relaxation time (Bass et al's model)

$$\frac{1}{\tau_{O_2}} = \frac{2\pi p}{p_0}\left(24 + 4.04\times10^4 h_a \frac{0.02 + h_a}{0.391 + h_a}\right)$$

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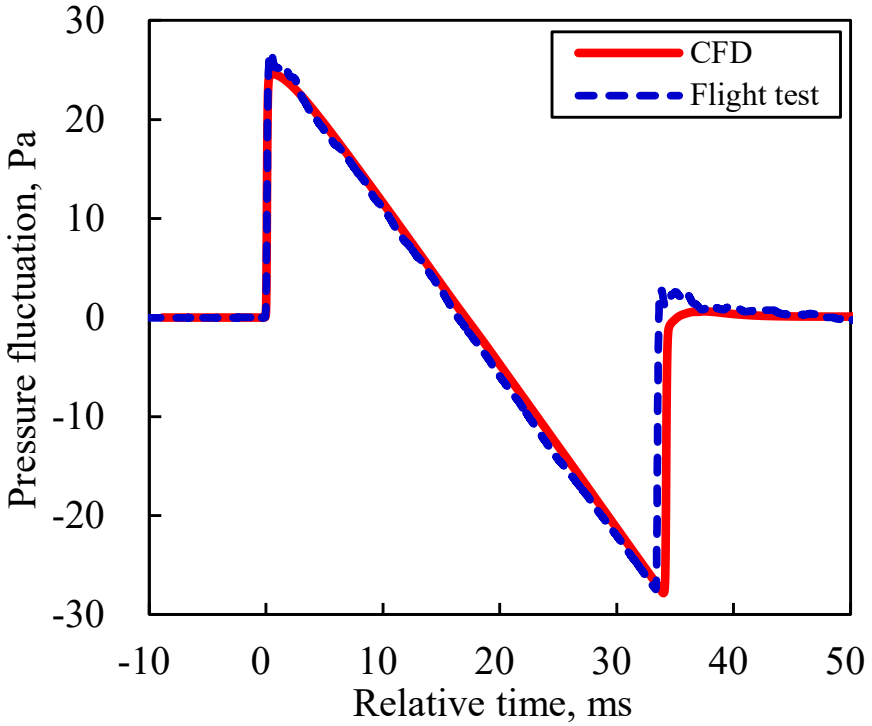
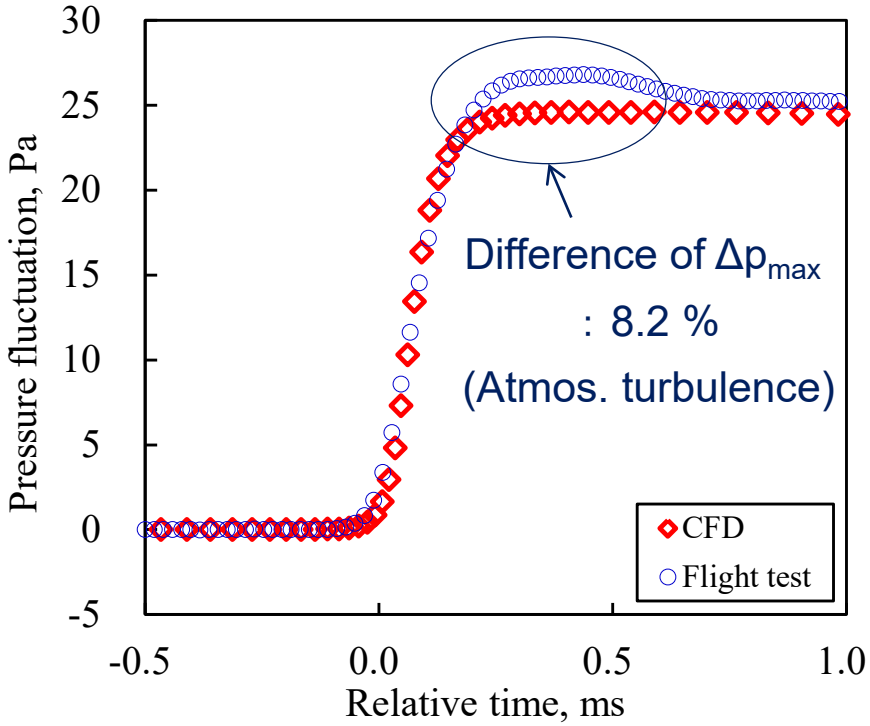
# 3. Simulation accuracy

Entire view




Closeup



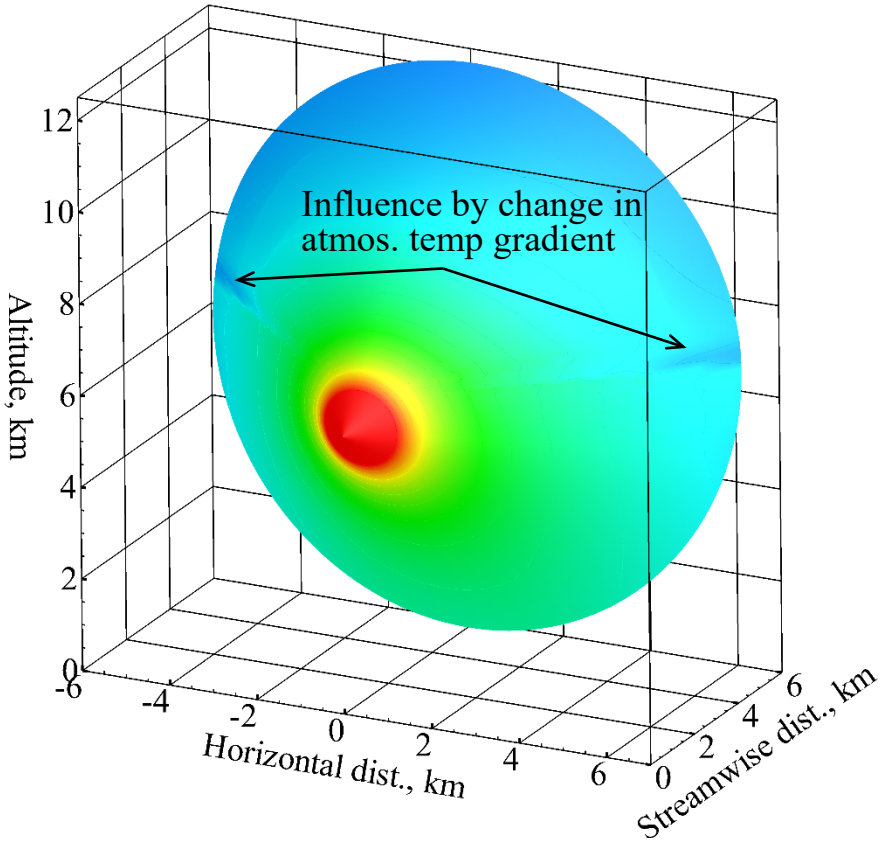

Pressure waveform（h = 0.5 km）

- Waveform shape is in good agreement with flight test data
- Rise time is well predicted considering relaxation effects

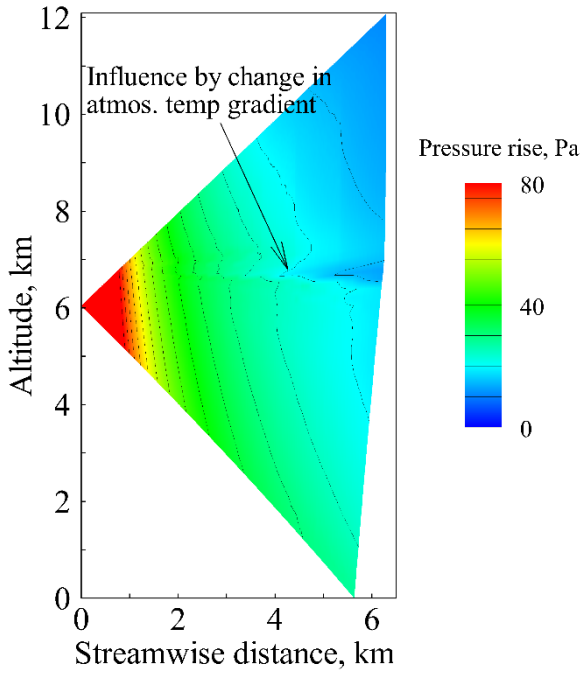
⇒ First ever direct reproduction of sonic boom was successful

# 3. Simulation accuracy

Three-quarter view




Side view




Pressure rise at front shock-wave surface

- Effect of atmospheric stratification is well considered
- 3D shock-wave structure is precisely captured

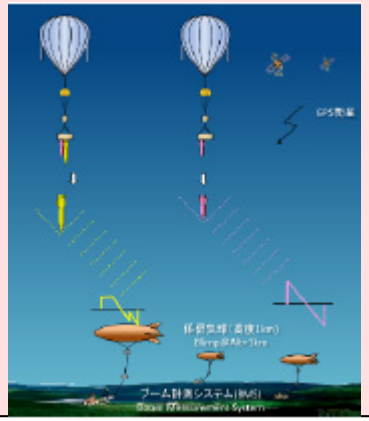
# Roadmap

**Formulation of simulation method**

- Computational method considering following five effects:
  - geometrical spreading
  - nonlinearity
  - molecular relaxation
  - atmos. stratification
  - viscosity
- Solution-adapted grid generation method
- Segmentation method of computational domain

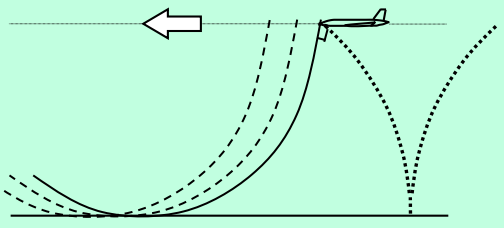
**Validation of full-field simulation method**

- Reproduction of flight test for sonic boom at supersonic speed
- Comparison with waveform parameter method

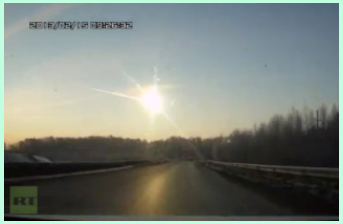


**Demonstration of applicability**

- Sonic boom cutoff phenomena at low supersonic speed



- Sonic boom propagation at hypersonic speed

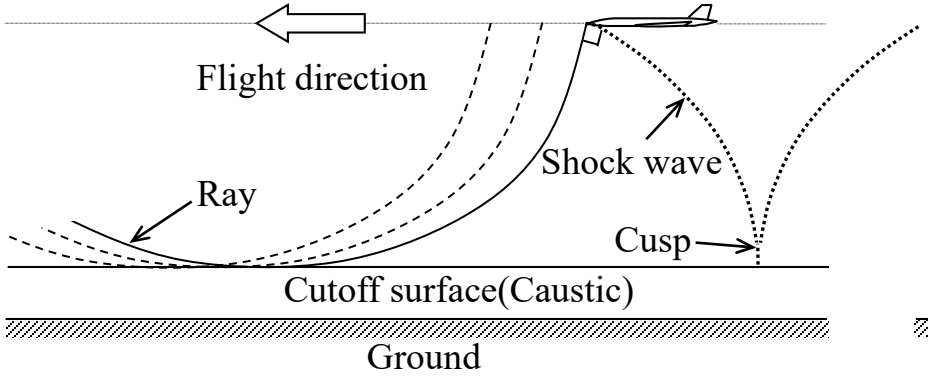


- Clarification of cutoff phenomena due to diffraction effect
- Demonstration of applicability to low supersonic flow analysis

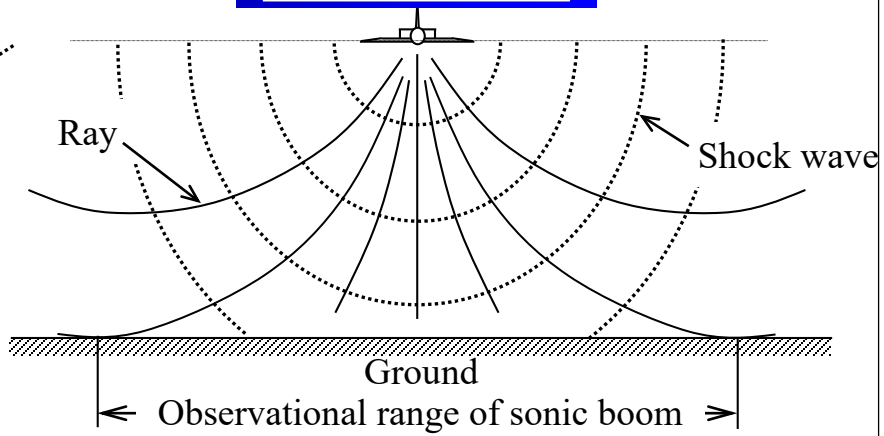

- Yamashita, R. et al, Transactions of JSASS, Vol.58, No.6, pp. 327-336, 2015.

# 4. Low-supersonic flow analysis

## Mach cutoff




## Lateral cutoff




Altitude decrease → Temperature increase → Local Mach number decrease → Mach angle increase

$$M = \frac{U}{\sqrt{\gamma RT}}$$

$$\theta = \sin^{-1}\frac{1}{M}$$

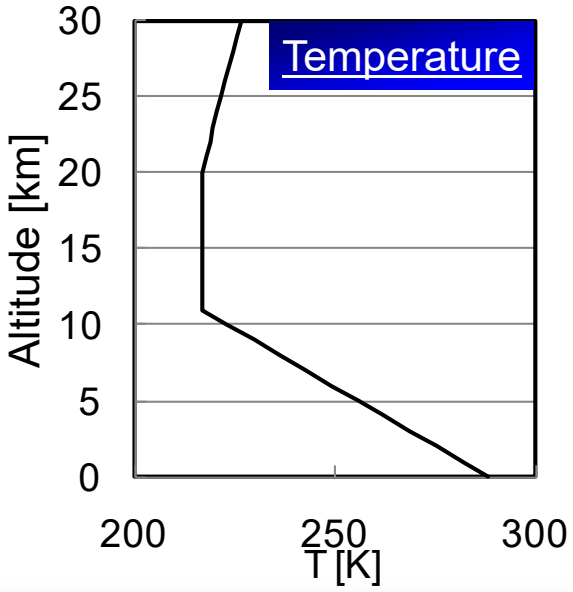
## Cutoff phenomena

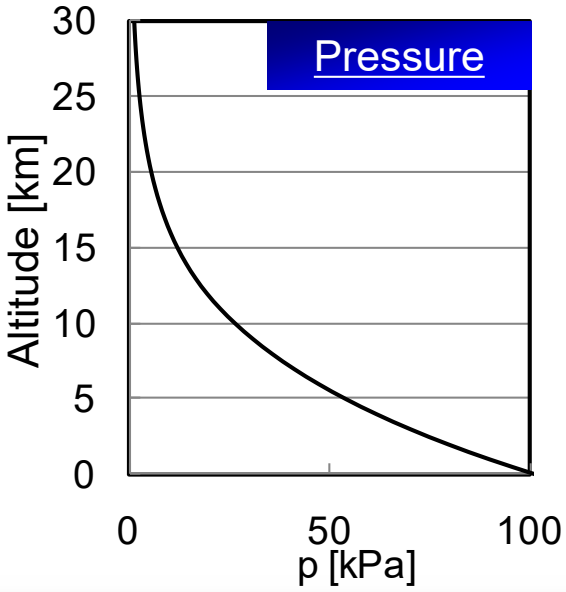
- Sonic boom does not reach the ground when local Mach number is less than 1 ( $1 < M_\infty < 1.2$ )
- Mach cutoff is difficult to evaluate by conventional methods

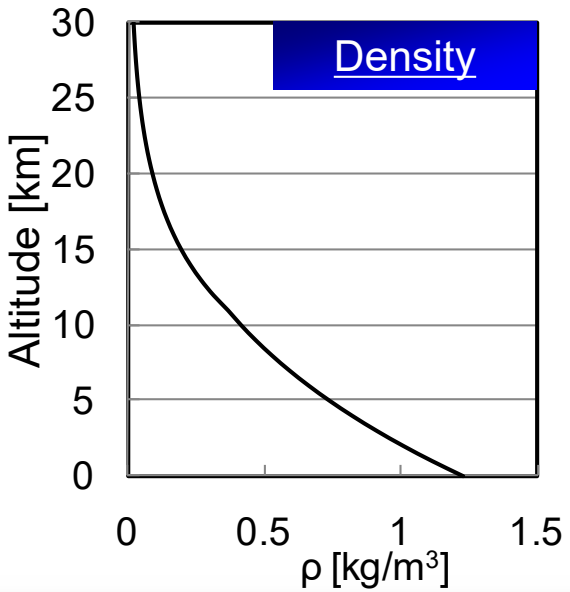
# 4. Low-supersonic flow analysis

## Numerical condition

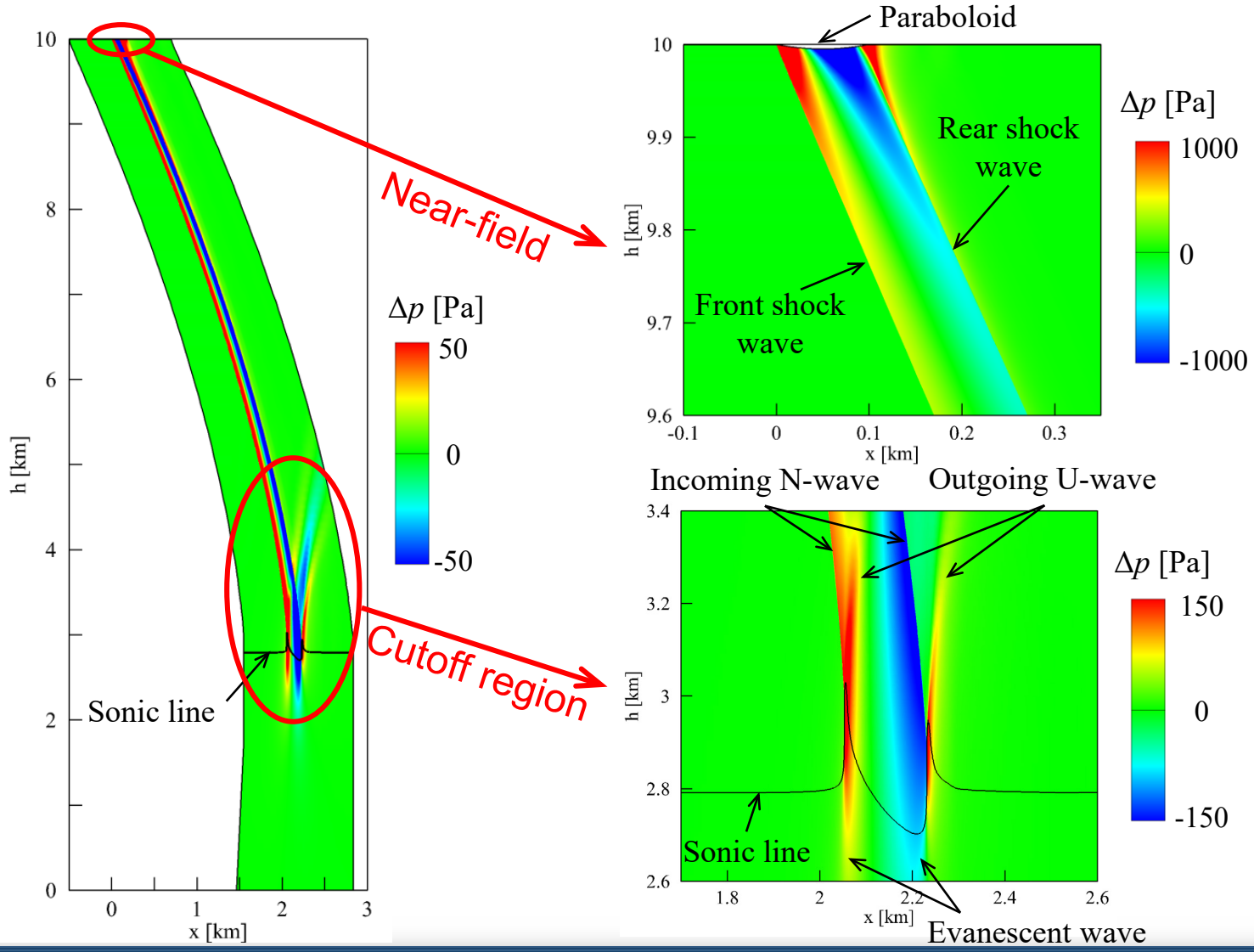
- Governing eq.: 3D Euler eq. + gravity term (thermal equilibrium)
- Flight model : Axi-symmetric paraboloid (L = 100 m, D/L = 0.1)
- Flight Mach number : $M_\infty$ = 1.1
- Flight altitude : h = 10 km
- Atmos. model : ISO standard atmosphere



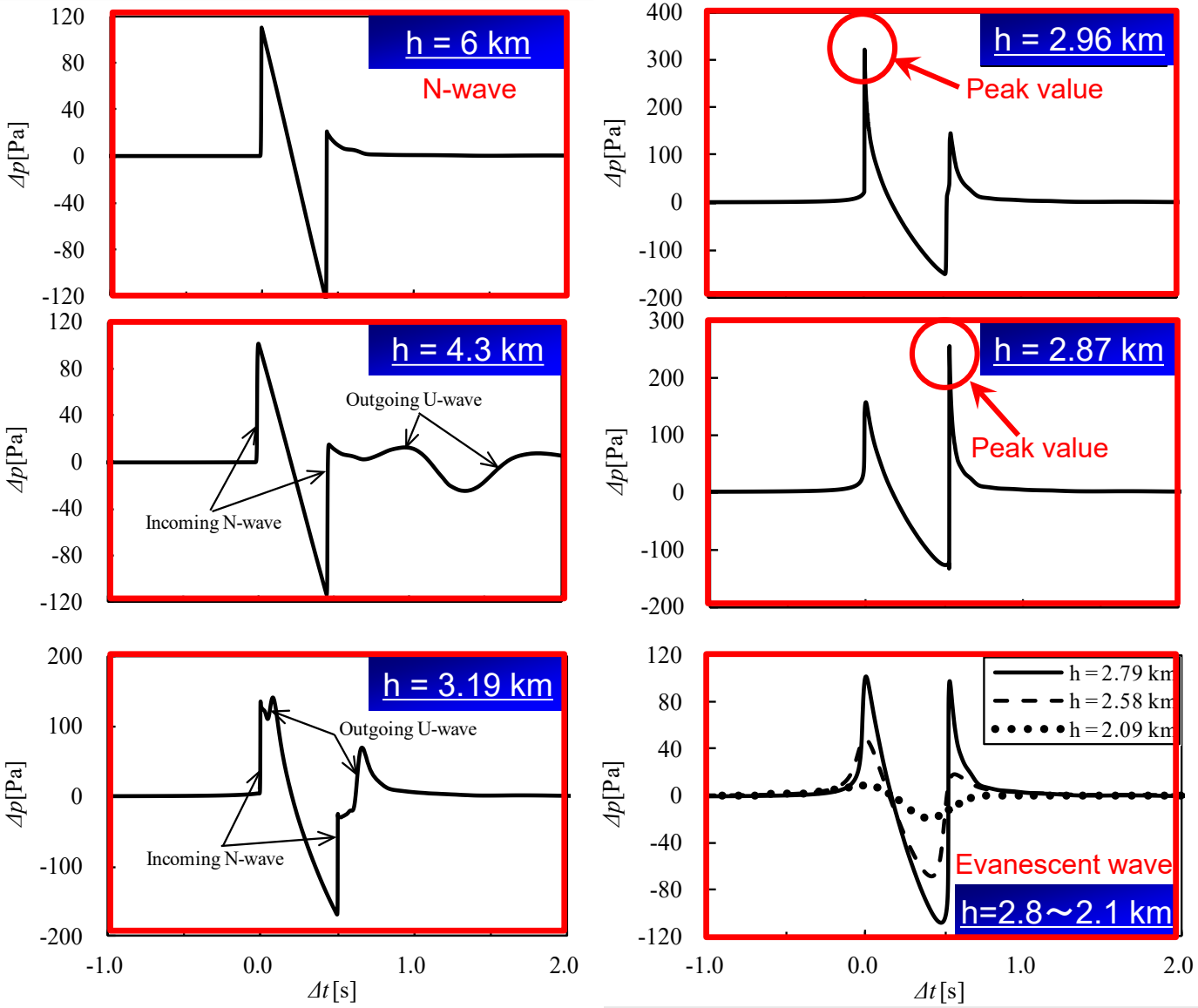



## Distribution of pressure rise on symmetry plane under body

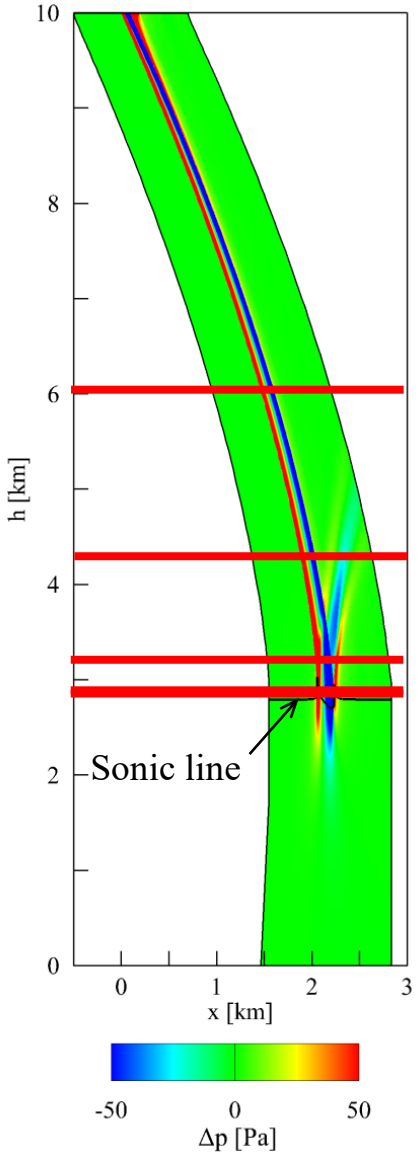


# 4. Low-supersonic flow analysis

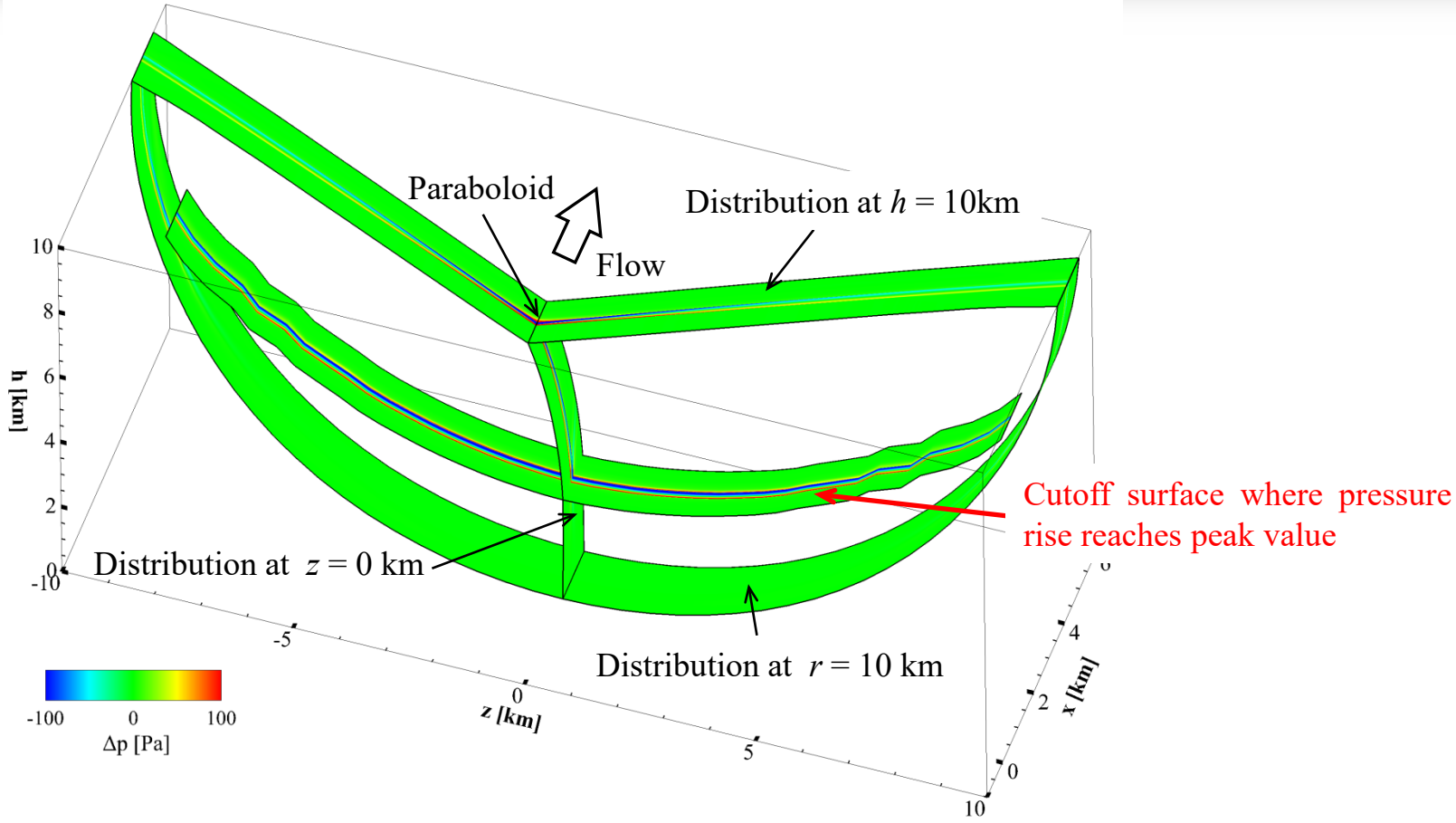








# 4. Low-supersonic flow analysis




- Cutoff surface is downward convex
- Mach cutoff altitude increases with lateral distance

⇒ 3D structure can be clarified by full-field simulation

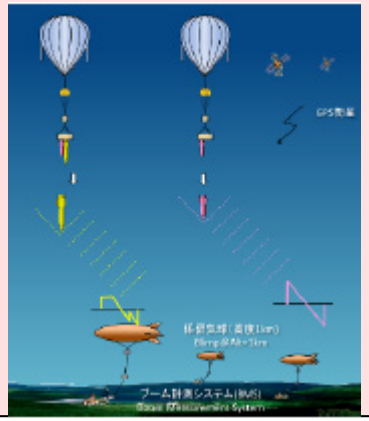
# Roadmap

**Formulation of simulation method**

- Computational method considering following five effects:
  - geometrical spreading
  - nonlinearity
  - molecular relaxation
  - atmos. stratification
  - viscosity
- Solution-adapted grid generation method
- Segmentation method of computational domain

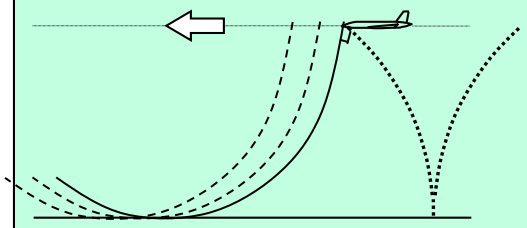
**Validation of full-field simulation method**

- Reproduction of flight test for sonic boom at supersonic speed
- Comparison with waveform parameter method

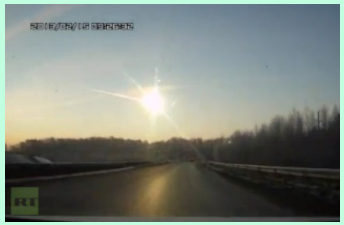


**Demonstration of applicability**

- Sonic boom cutoff phenomena at low supersonic speed



- ☐ Sonic boom propagation at hypersonic speed



Chelyabinsk meteorite：The number of fragments was same as that of sounds

⇒ Waveform shape might not be typical N-wave

⇒ Effect of Mach number on waveform is investigated

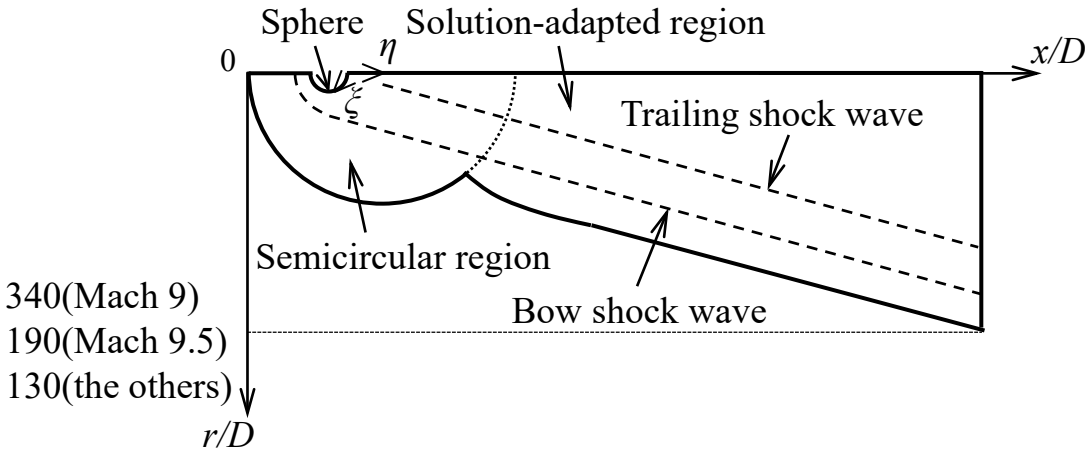
・Yamashita R. et al., AIAA Journal, Vol. 54, No. 2, pp. 767-770, 2016.

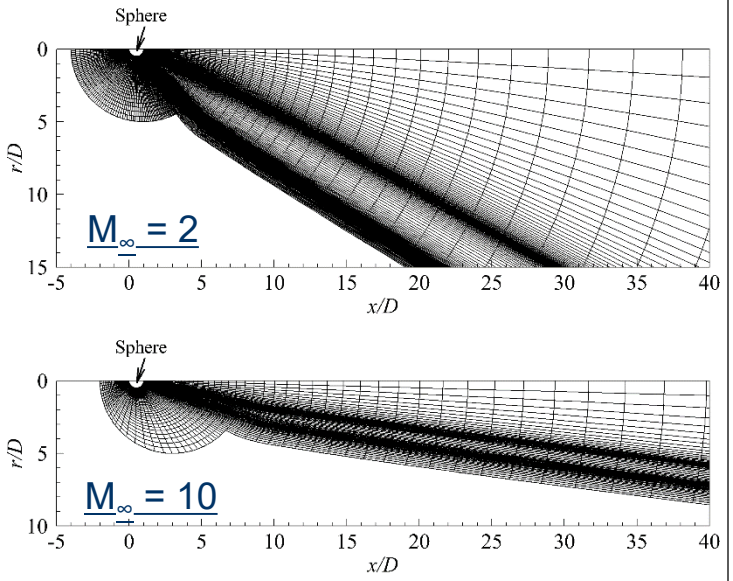
# 5. Hypersonic flow analysis

## Numerical condition

- Governing eq. : Axi-symmetric Navier-Stokes eq.
- Flight object : Sphere（D = 20 m, steady level flight）
- Flight Mach number : $M_\infty$ = 2(supersonic), 5～11（hypersonic）
- Atmosphere : Geometric mean of properties at 25 and 0 km alt.

## Numerical grid

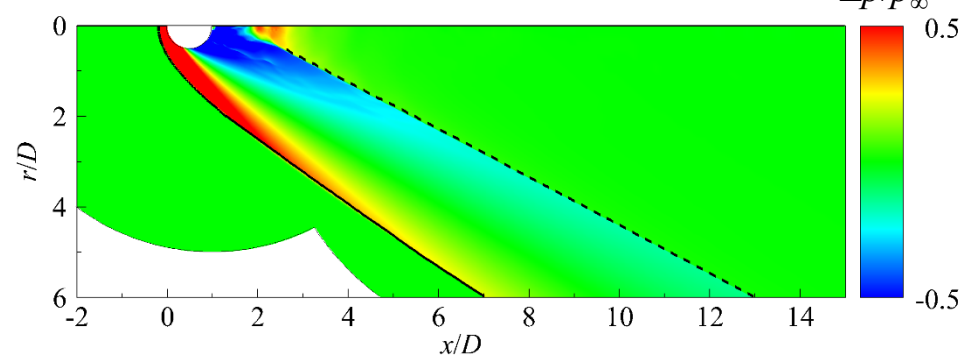


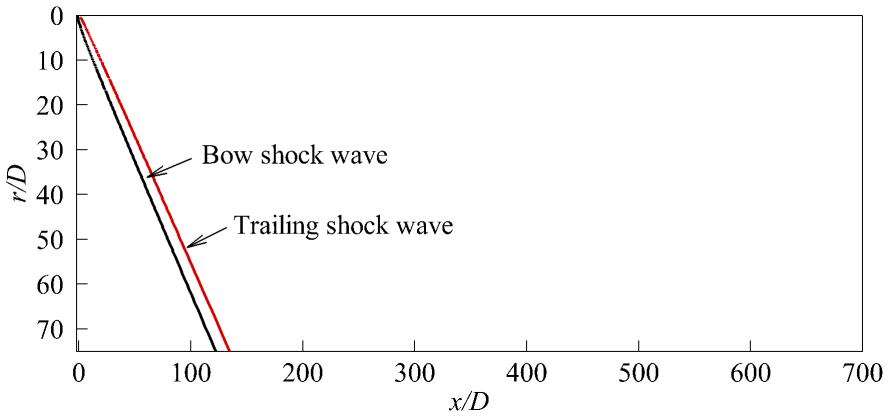




- Number of grid points : 0.27～0.29 million

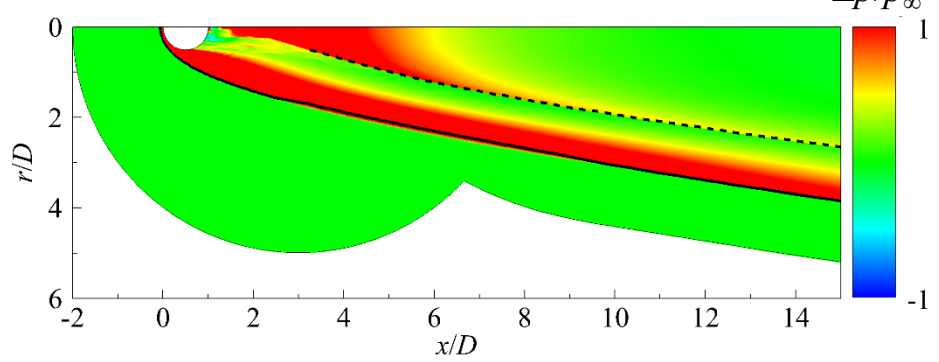
# 5. Hypersonic flow analysis

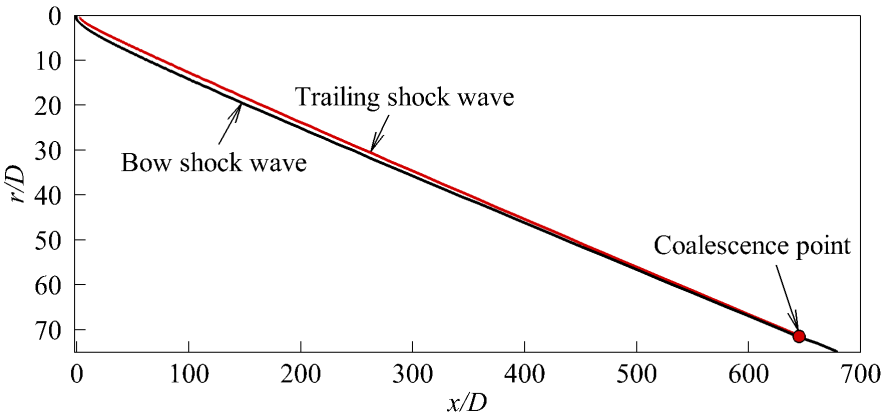
$M_\infty = 2$



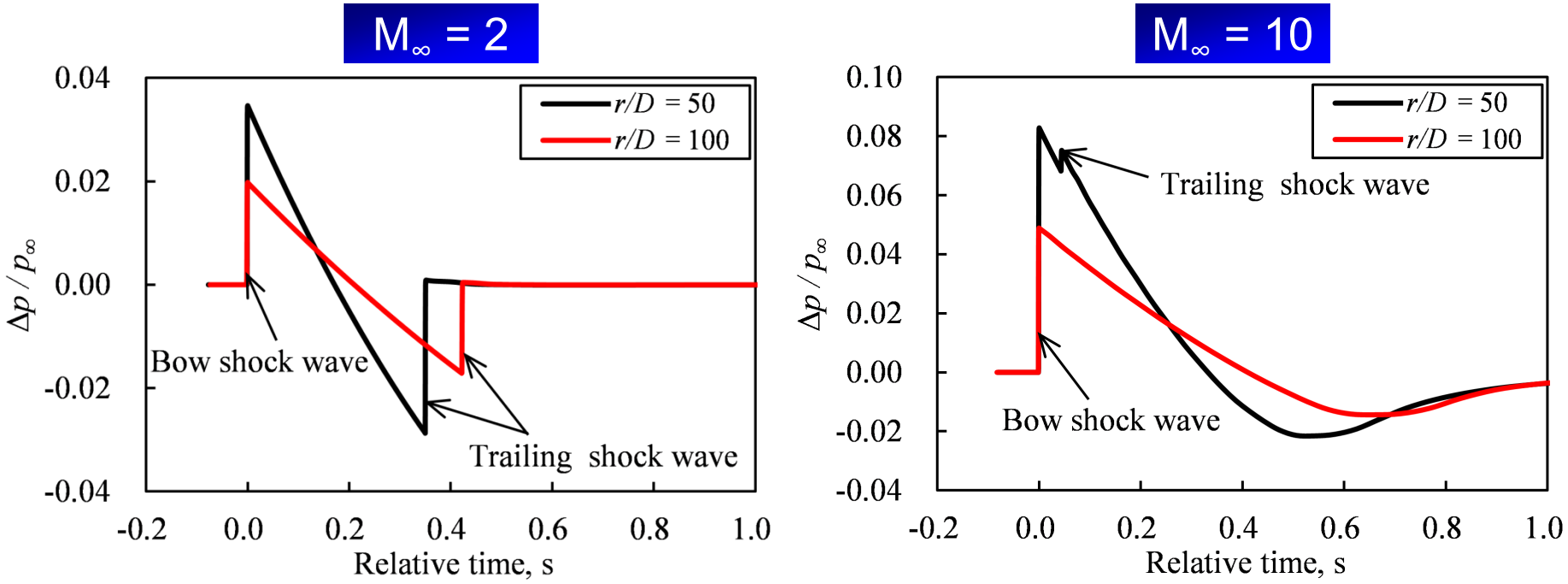

$M_\infty = 10$




- Location of shock wave is defined as that at local max. pressure
- $M_\infty = 2$ : Bow (BSW) and trailing (TSW) shock waves separate
- $M_\infty = 10$ : 〃 coalesce

# 5. Hypersonic flow analysis



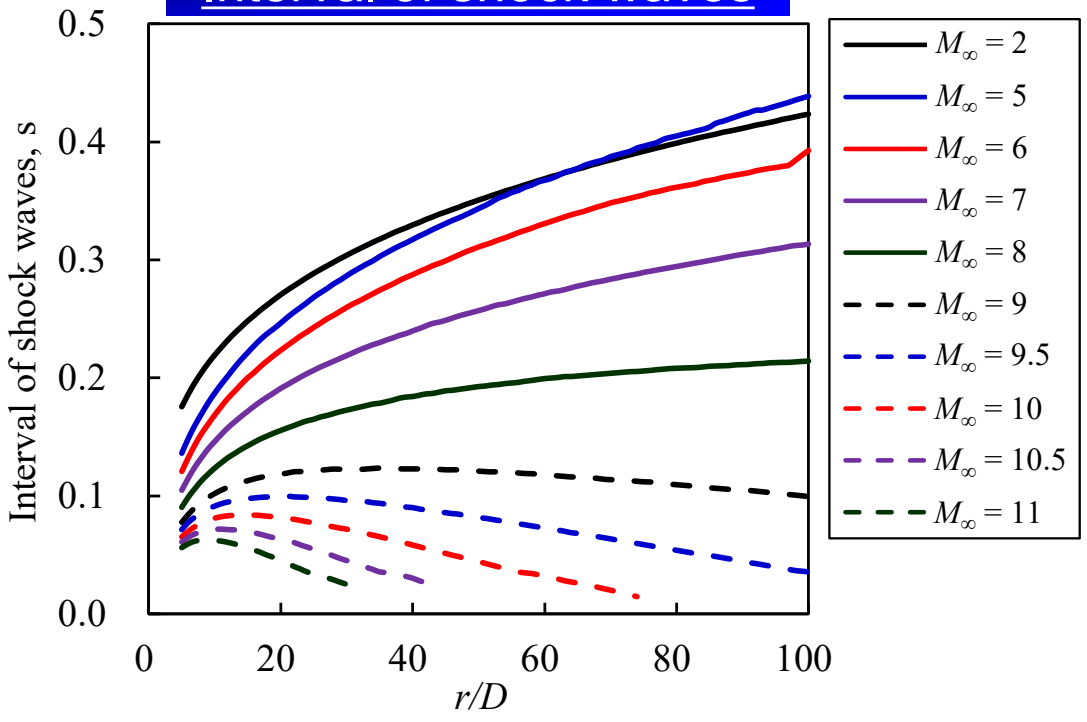

## Far-field pressure waveform

- Mach 2 : N-wave（explosive sound occurs twice）
- Mach 10 : Caret-wave（explosive sound occurs once）

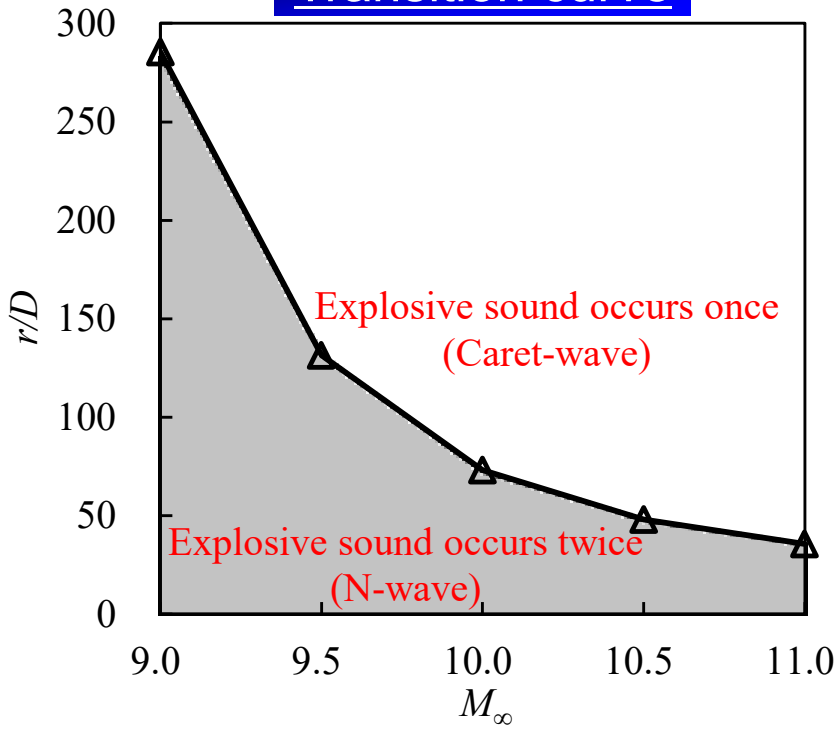
Waveform type is changed as flight Mach number increases

# 5. Hypersonic flow analysis

## Interval of shock waves




## Transition curve



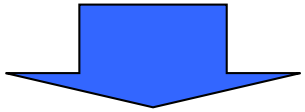

- Interval of shock waves（Near field）：Increase in all cases
- 〃 （Far field）：increase（$M_\infty \leq 8$）, decrease（$M_\infty \geq 9$）

⇒ Transition Mach number exists in $8 \leq M_\infty \leq 9$

- Explosive sound occurs once for caret-wave when $M_\infty \geq 9$

# 6. Conclusion

- First ever direct simulation of sonic boom was successfully performed by full-field simulation
- The following sonic boom phenomena were well clarified
  - Cutoff phenomena at low supersonic speed
  - Waveform transition at hypersonic speed



Full-field simulation is promising for realizing numerical flight experiment

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3. Yamashita R. and Suzuki K. Rise Time Prediction of Sonic Boom by Full-Field Simulation. Journal of Aircraft, Vol. 55, No. 3, pp. 1305-1310, 2018.
4. Yamashita R. and Suzuki K. Full-Field Simulation for Sonic Boom Cutoff Phenomena. Transactions of the Japan Society for Aeronautical and Space Sciences, Vol. 58, No. 6, pp. 327-336, 2015.
5. Yamashita R. and Suzuki K. Waveform Transition of Sonic Boom Generated from a Sphere at Hypersonic Speed. AIAA Journal, Vol. 54, No. 2, pp. 767-770, 2016.
6. Yamashita R. and Suzuki K. Sonic Boom Analysis for Hypersonic Vehicle by Global Direct Simulation, Journal of the Japan Society for Aeronautical and Space Sciences, Vol. 62, No. 3, pp. 77-84, 2014 (in Japanese).

Thank you for your kind attention !