POLITECNICO DI MILANO

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Reliability assessment of buckling response of axially compressed sandwich composite shells with and without cut-outs

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Outline

- Introduction
- Shell Description and Finite Element Model
- Probabilistic Procedure for Buckling Analysis
- Results of Probabilistic Procedure
- Conclusions

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Introduction

Within the running EU project **DESICOS** (New Robust DESign Guideline for Imperfection Sensitive COmposite Launcher Structures), a **Probabilistic Methodology** was developed for **Reliability Buckling Analysis** of cylindrical shells.

The scaled models of the Dual Launch System (**SYLDA**) and of the Interstage Skirt Structure (**ISS**) of Ariane 5 launcher were probabilistically investigated:

- The SYLDA model is also investigated with three circular cut-outs (**SYLDA with cut-outs**).
- The scaled models of SYLDA, SYLDA with cut-outs and ISS were designed by Airbus Defence & Space.
- The three structures are sandwich composite shells made of the same material, but with different stacking sequence and geometric dimensions.
- A loading condition of pure compression is assumed.

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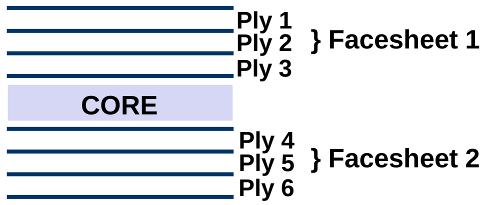
Shell Description

Material Data:

Ply Hexcel IM7/8552 UD carbon prepreg properties[1,2]

E_{xx} [MPa]	E_{yy} [MPa]	G_{xy} [MPa]	ν_{xy}	ρ [kg/m^3]	t_{ply} [mm]
150000	9080	5290	0.32	1570	0.131

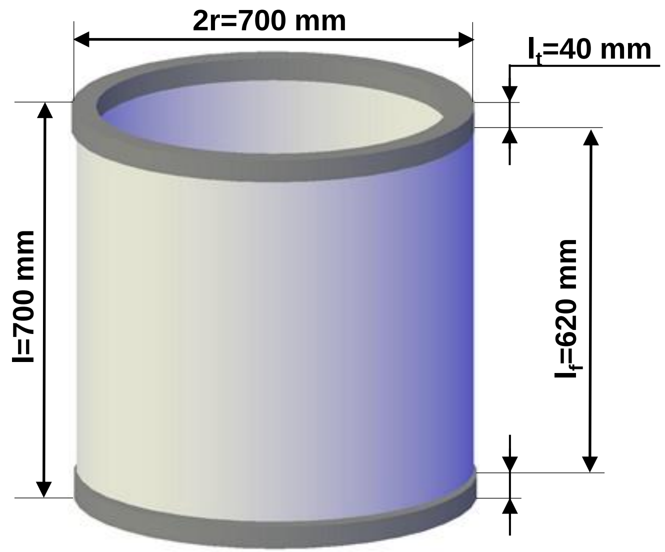


Core EVONIK Rohacell WF200 properties[3]

E_{xx} [MPa]	ν	ρ [kg/m^3]
350	0.3	205

SYLDA Shell[2]:

Layup: [19°/-19°/ 90°/CORE/ 90°/-19°/19°]
Core thickness: 1.5 mm
Total thickness: 2.286 mm

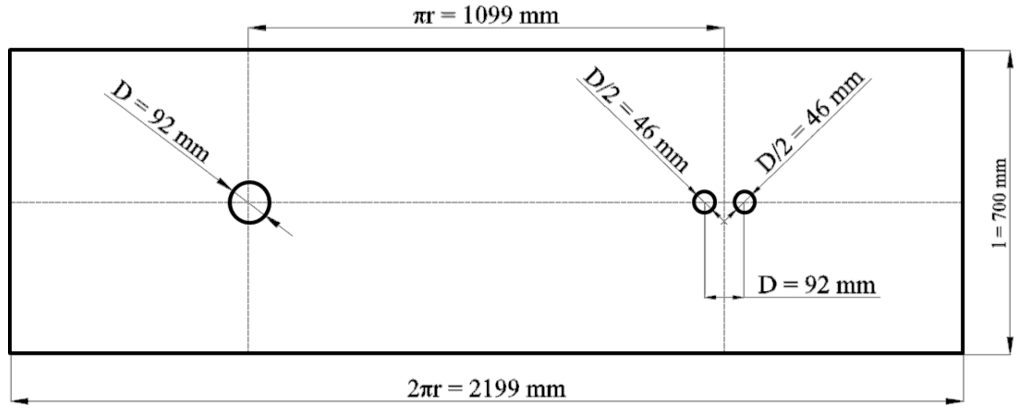


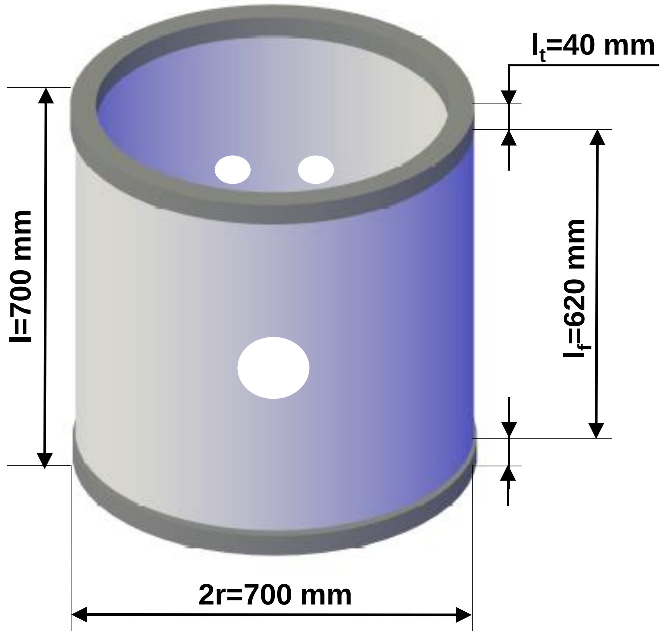
1. C. Bisagni, R. Vescovini and C. G. Dávila, Single-stringer compression specimen for the assessment of damage tolerance of postbuckled structures, *Journal of Aircraft*, 48(2) (2011) 495-502.
2. Alexandre, C. and Blanchard, P., "Definition of the reduced model - ASTRIUM - F Task," Release 1/1, 2013.
3. http://www.rohacell.com/sites/dc/Downloadcenter/Evonik/Product/ROHACELL/product-information/ROHACELL%20WF%20Product%20Information.pdf

Shell Description

SYLDA Shell with cut-outs:

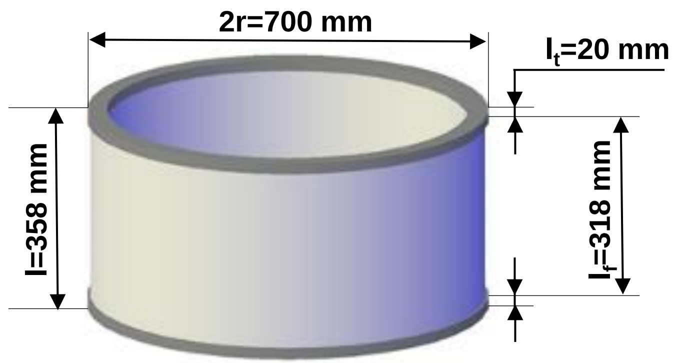
Layup: [19°/-19°/ 90°/CORE/ 90°/-19°/19°]
Core thickness: 1.5 mm
Total thickness: 2.286 mm





ISS Shell:

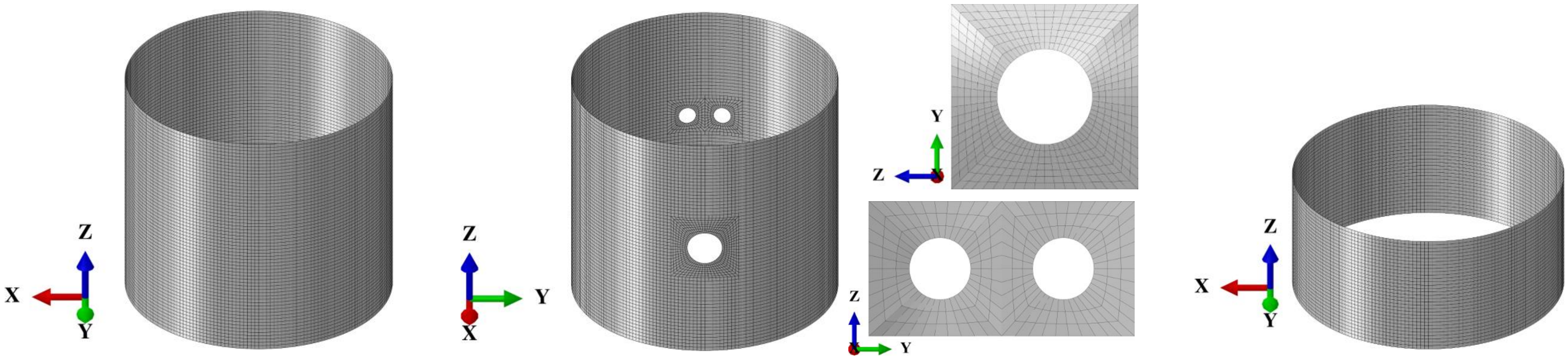
Layup: [30°/-30°/0°/CORE/0°/-30°/30°]
Core thickness: 2.6 mm
Total thickness: 3.386 mm



Finite Element Model

The **Finite Element Model** of each shell is set up using the commercial FE code ABAQUS ver. 6.13:

Configuration	Element type	Total number of elements
SYLDA	S4R	13640
SYLDA with cut-outs	S4R	13860
ISS	S4R	7040



SYLDA SYLDA with cut-outs ISS

Boundary conditions:

- Lower edge: clamped.
- Upper edge: clamped, but free to translate along the axial direction.

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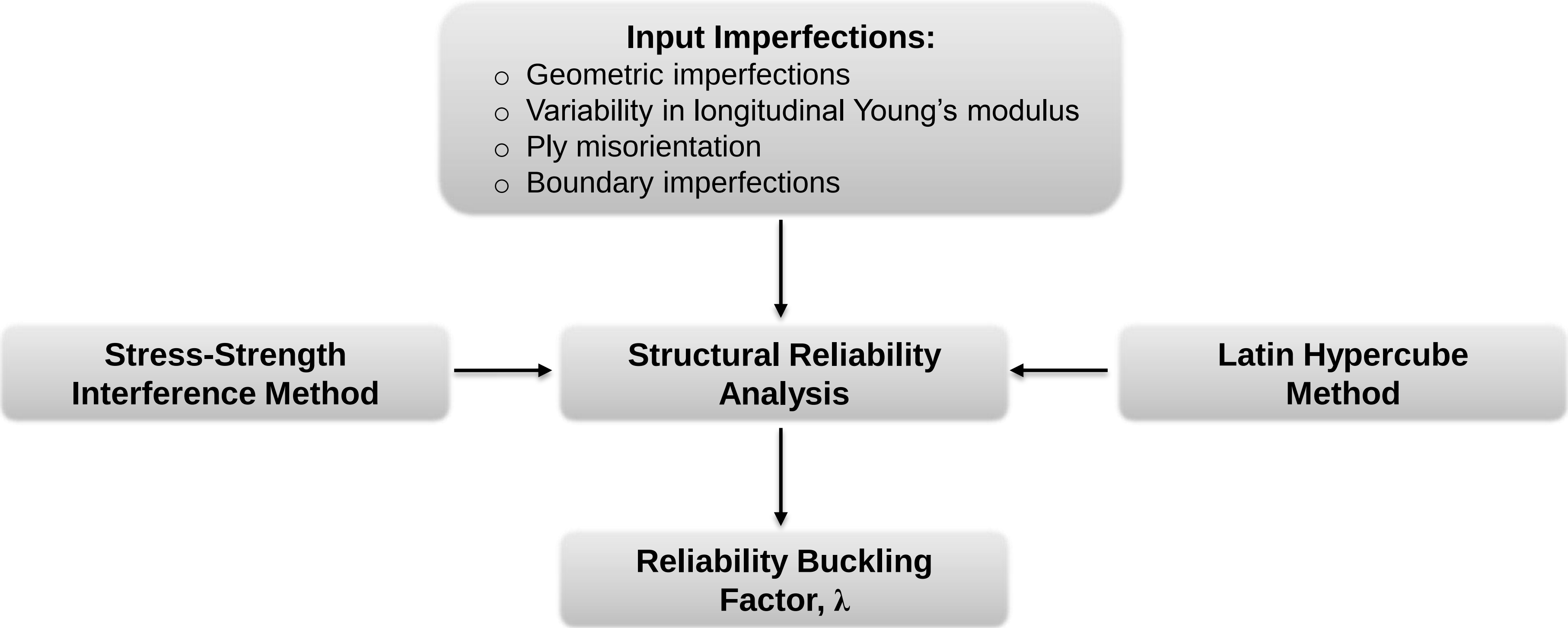
Probabilistic Procedure for Buckling Analysis

The Stress-Strength Interference Method and the Latin Hypercube Method are combined to perform a **Structural Reliability Analysis** of axially compressed cylindrical shells considering different types of input imperfections.



The goal is to determine the **Reliability Buckling Factor** λ for a probability level equal to **99%**.

Probabilistic Properties of Input Parameters

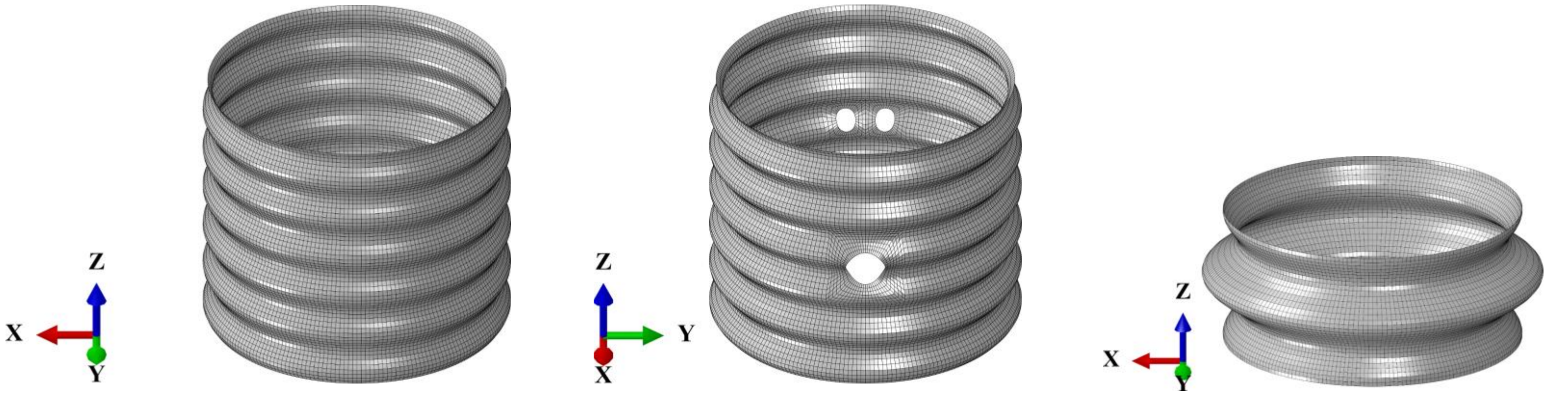
Probabilistic Properties of Input Parameters → Stress-Strength Interference Method → Latin Hypercube Method and Distribution Identification → Assessment of Reliability Buckling Factor

The **Geometric Imperfections** are assumed to have an axisymmetric shape modeled by the function:

$$\frac{w}{t} = \xi \sin\left(i\pi \frac{z}{l}\right)$$

i, number of axial half-waves;
z, axial coordinate;
ξ, imperfection amplitude assumed normally distributed[4]:
$\xi \sim$**N(**$\bar{\xi}$, s_ξ**)=N(-0.0083, 0.0316)**.

Amplified geometric imperfections:



SYLDA **SYLDA with cut-outs** **ISS**

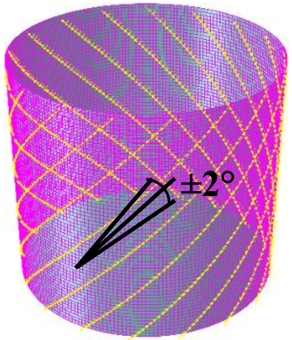
To account for **Variability in Longitudinal Young's Modulus**, it is assumed to have a Gaussian distribution[5]: $E_{11}\sim$**N(**$\bar{E}_{11}$, $s_{E_{11}}$**) = N(171.42 GPa, 6.84 GPa)**.

4. J. Arbocz and M. W. Hilburger, Toward a probabilistic preliminary design criterion for buckling critical composite shells, *AIAA Journal*, 43(8) (2005) 1823-1827.
5. P. P. Camanho, P. Maimí and C. G. Dávila, Prediction of size effects in notched laminates using continuum damage mechanics, *Composite Science and Technology*, 67 (2007) 2715-2727.

Probabilistic Properties of Input Parameters

Probabilistic Properties of Input Parameters → Stress-Strength Interference Method → Latin Hypercube Method and Distribution Identification → Assessment of Reliability Buckling Factor

In order to capture **Ply Misoreintation,** the orientation of each lamina is assumed to be independent on each other and normally distributed with a tolerance of $\pm 2°$, typical in the aerospace field.

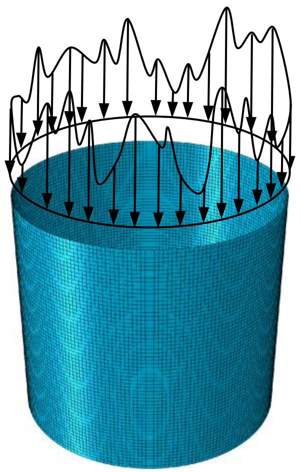


The **Boundary Imperfections** are modeled using a one-dimensional homogeneous Gaussian field:

Mean value is the nominal magnitude of applied axial displacement.
Covariance Σ is modeled by the exponential function[6]:

$$\Sigma = \Sigma_0 \exp\left(-\frac{|d|}{l_0}\right)$$

d, circumferential distance between two nodes of loaded edge;
l_0, correlation length set to πr.



The variance Σ_0 is set to have the resulting profile of reaction forces with coefficient of variation equal to **15%**[7].

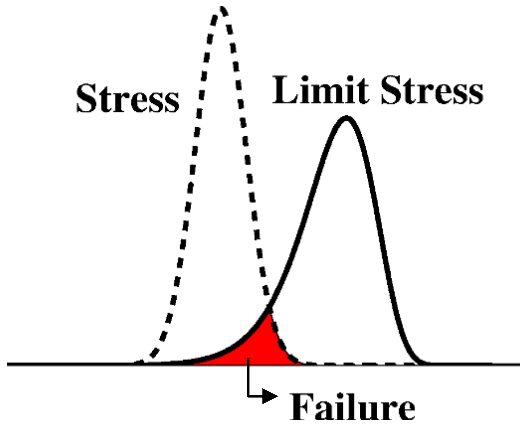
6. S. K. Choi, R. V. Grandhi, R. A. Canfield, *Reliability-Based Structural Design* (Springer-Verlag, 2007).
7. T. De Mollerat, C. Vidal and M. Klein, Reliability based factor of safety for unmanned spacecrafts, in *Structural Safety Evaluation Based on System Identification Approaches*, eds. H. G. Natke and J. T. P. Yao (Springer, 1988), pp. 266-312.

Stress-Strength Interference Method

Probabilistic Properties of Input Parameters → **Stress-Strength Interference Method** → Latin Hypercube Method and Distribution Identification → Assessment of Reliability Buckling Factor

- The **Limit State Function g(X)** or **Margin of Safety M** is defined as:

$$M = g(X) = \text{Limit stress} - \text{Stress} = \Lambda_S(X) - \lambda$$



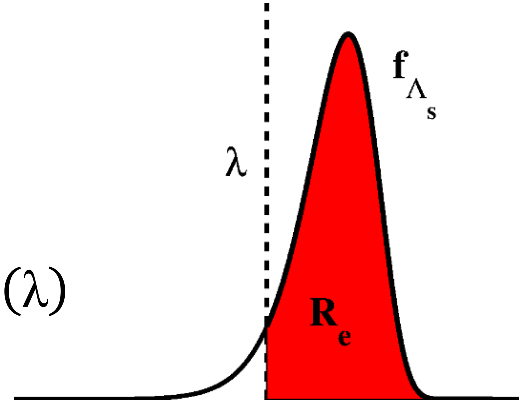
The **Normalized Buckling Load Λ_s(X)** is a random variable characterized through its probability distribution function.

The vector **X** includes the input random parameters.

The normalized loading parameter, named **Reliability Buckling Factor** λ, is a deterministic variable.

- The **Reliability R_e** is defined as the probability that M > 0:

$$R_e = \text{Prob}(M > 0) = \text{Prob}(\Lambda_S(X) - \lambda > 0) = \int_{\lambda}^{+\infty} f_{\Lambda_S}(\Lambda_S)\, d\Lambda_S = 1 - F_{\Lambda_S}(\lambda)$$



- Thus, once specified the requirement of reliability level and identified the probability distribution function of Λ_s(X), the corresponding λ is assessed.

Latin Hypercube Method and Distribution Identification

Probabilistic Properties of Input Parameters → Stress-Strength Interference Method → **Latin Hypercube Method and Distribution Identification** → Assessment of Reliability Buckling Factor

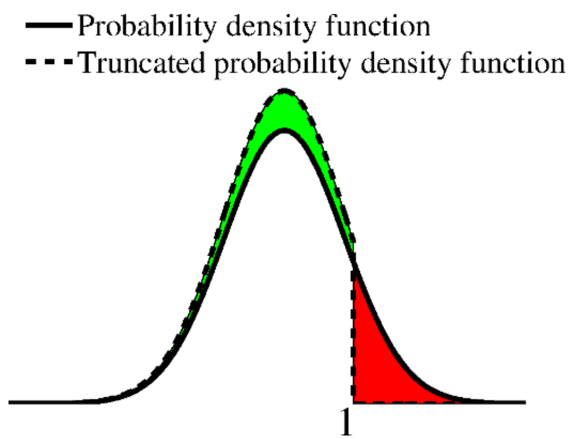
- The **Latin Hypercube Method** is applied to generate a data set of $\Lambda_s(X)$.

  Sample size **N=100** is chosen in order to keep a limited computational time.

  The obtained data collection of buckling load is normalized by the buckling load of shell considering the **Nominal Values** of input probabilistic parameters.

- The data collection is used to identify the statistical characteristics and the distribution of $\Lambda_s(X)$ by means of:
  - **Goodness-of-fit Statistics** (Quantile-Quantile Plot, Kolmogorov-Smirnov test, Anderson-Darling test).
  - **Histogram** and **Statistical Estimators**.

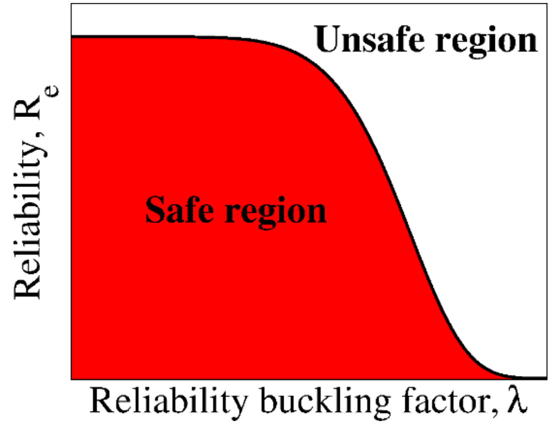
- The distribution of $\Lambda_s(X)$ is **Truncated From Above** so that the reliability buckling factor cannot assume values higher than one.



Assessment of Reliability Buckling Factor

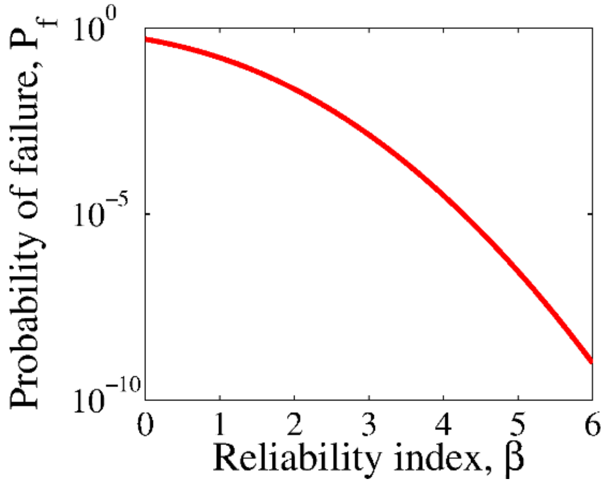
Probabilistic Properties of Input Parameters → Stress-Strength Interference Method → Latin Hypercube Method and Distribution Identification → **Assessment of Reliability Buckling Factor**

- The reliability buckling factor λ is estimated using the inverse cumulative density function $F^{-1}_{\Lambda_s}$ of $\Lambda_s(X)$, for a reliability level equal to 99%:



$$R_e = 1 - F_{\Lambda_s}(\lambda) \xrightarrow{\text{yields}} \lambda = F^{-1}_{\Lambda_s}(1 - R_e)$$

- In case that $\Lambda_s(X)$ is normally distributed and is uncorrelated from λ, the equation is re-formulated using the **Reliability Index**[6] β and the relationship $\mathbf{R_e = \Phi(\beta)}$[6] :



$$R_e = 1 - F_{\Lambda_s}(\lambda) \xrightarrow{\text{yields}} \lambda = \overline{\Lambda}_s + \beta \cdot s_{\Lambda_s}$$

Φ is the standard normal cumulative density function.

6. S. K. Choi, R. V. Grandhi, R. A. Canfield, *Reliability-Based Structural Design* (Springer-Verlag, 2007).

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Probabilistic Buckling Analysis of SYLDA

The reliability buckling factor λ of SYLDA is estimated for a reliability level equal to **99%** in the different analysis cases.

The NASA knockdown factor[8] is determined on the assumption that SYLDA can be considered a laminated composite shell of seven plies.

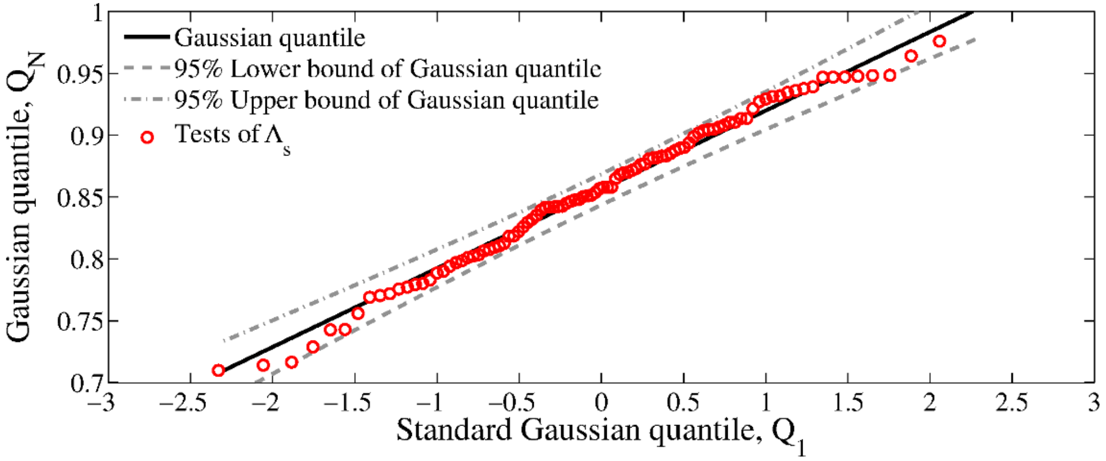
Analysis case	Reliability Buckling factor, λ	Load [kN]
Analysis of nominally perfect shell	1	459
Analysis with geometric imperfections	0.73	335
Analysis with variability in longitudinal Young's modulus	0.92	422
Analysis with ply misoreintation	0.97	445
Analysis with boundary imperfections	0.89	409
Analysis with combined imperfections	0.71	326
NASA knockdown factor[8]	0.57	262

Out of all imperfections, the geometric imperfections are more dominant in determining the buckling response of SYLDA.

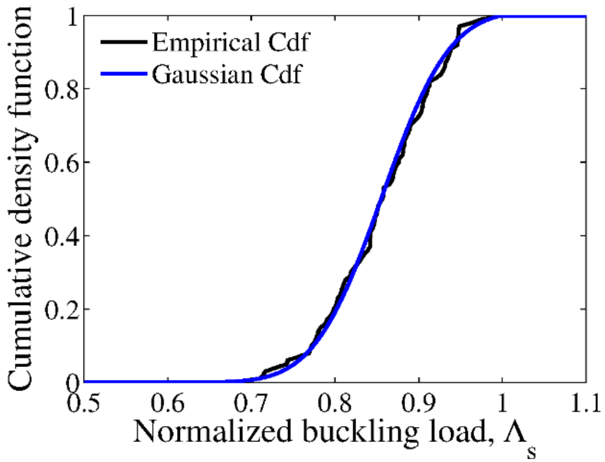
8. V. I. Weingarten, P. Seide, and J. P. Peterson, NASA SP-8007 - Buckling of Thin-Walled Circular Cylinders, National Aeronautics and Space Administration, Washington, DC, USA (1968).

Probabilistic Buckling Analysis of SYLDA

Quantile-Quantile Plot



Kolmogorov–Smirnov Test



Anderson-Darling Test:

1. Calculate test statistic:

$$AD=-N-\sum_{i=1}^{N}\frac{2i-1}{N}\left[\ln F_{\Lambda_s}\left(\Lambda_{s,i}\right)-\ln\left(1-F_{\Lambda_s}\left(\Lambda_{s,N+1-i}\right)\right)\right]=0.30$$

2. Calculate modified test statistic to account for sample size:

$$AD^* = AD\left(1+\frac{0.2}{\sqrt{N}}\right) = 0.30$$

3. Calculate observed significance level:

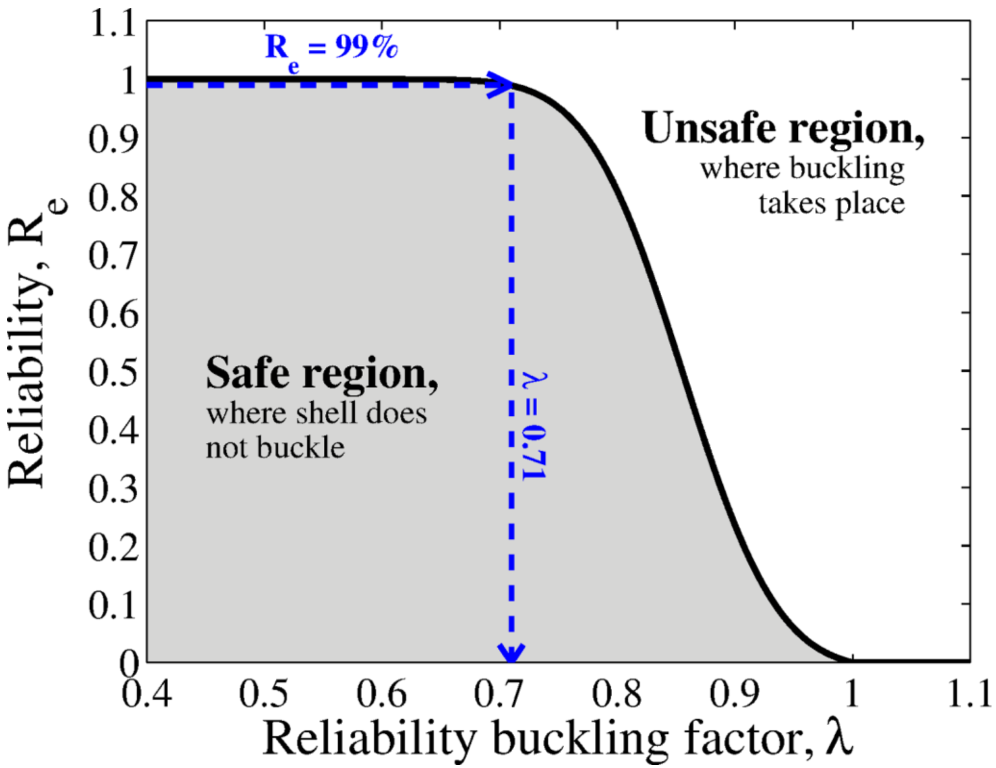
$$OSL = \left[1+\exp\left(-0.48 + 0.78\ln\left(AD^*\right) + 4.58AD^*\right)\right]^{-1}=0.51$$

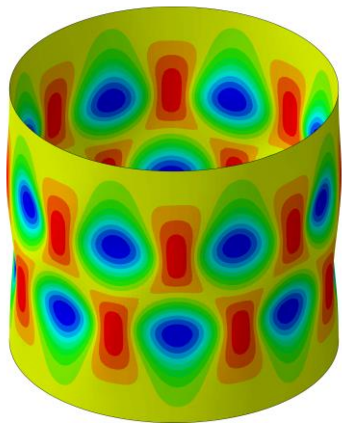
4. Since OSL > α=0.05 (95% confidence level), the assumption of Gaussian distribution is not rejected.

As consequence of the assumed uncertainties about model parameters, the $\Lambda_s(X)$ of SYLDA combining all sources of input imperfections is accepted to have a **Gaussian Distribution**.

Probabilistic Buckling Analysis of SYLDA

The reliability **$R_e(\lambda)$** of SYLDA combining all sources of input imperfections is determined as function of the reliability buckling factor λ:





Post-buckling shape of nominally perfect SYLDA at δ =2.50 mm.

The value of buckling load corresponding to $\boldsymbol{\lambda=0.71}$ is equal to **326 kN**.

Probabilistic Buckling Analysis of SYLDA with Cut-outs

The data collection of $\Lambda_s(X)$ is obtained through dividing the **Maximum Load** reached by the shell with imperfections by the maximum load reached by the nominally perfect shell.

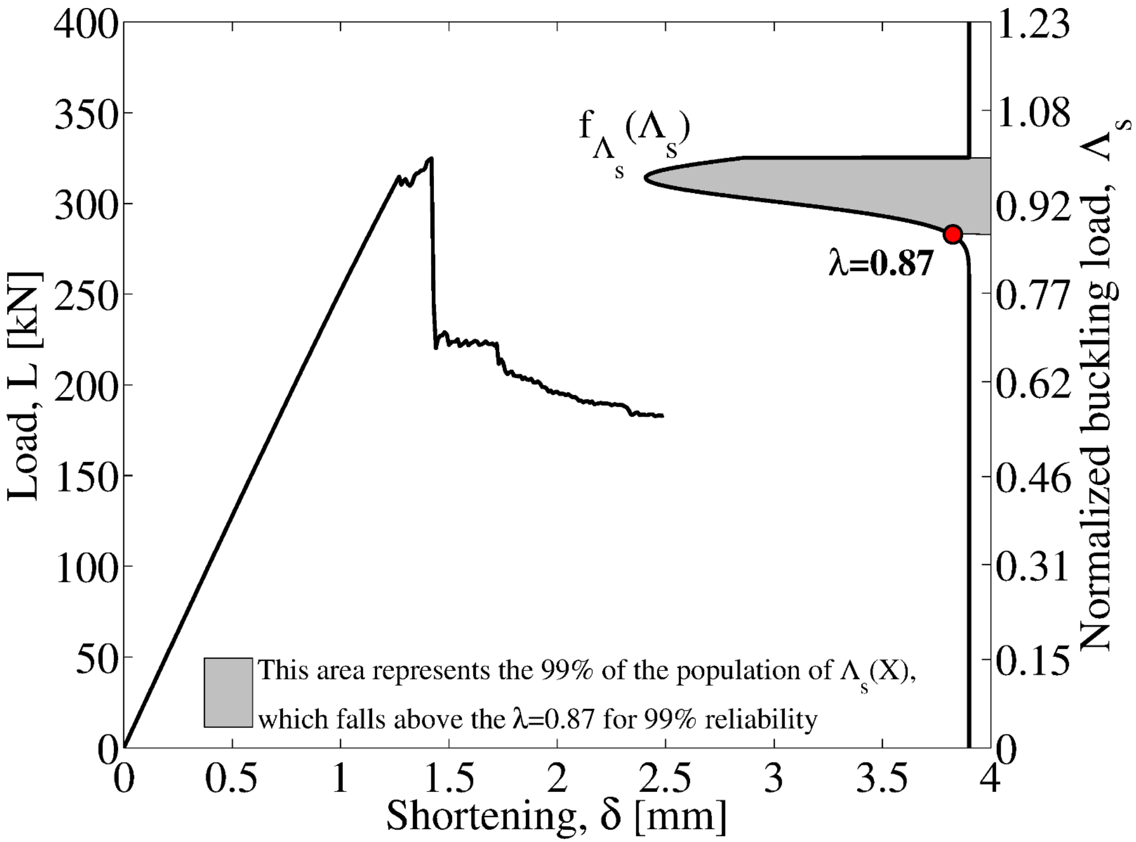
The reliability buckling factor λ of SYLDA with cut-outs is assessed in the different analysis cases for a probability of **99%**.

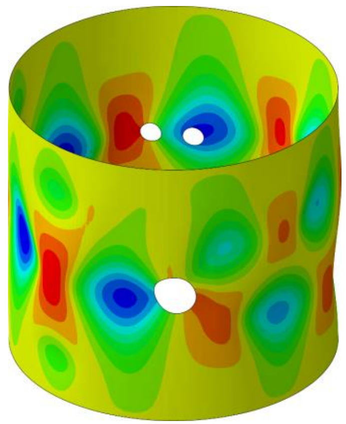
Analysis case	Reliability Buckling factor, λ	Load [kN]
Analysis of nominally perfect shell	1	325
Analysis with geometric imperfections	0.90	293
Analysis with variability in longitudinal Young's modulus	0.93	302
Analysis with ply misoreintation	0.97	315
Analysis with boundary imperfections	0.94	306
Analysis with combined imperfections	0.87	283
NASA knockdown factor	N/A	N/A

The value of λ is not dominated by a specific source of imperfections, but it is caused by the combined influence of all input probabilistic parameters.

Probabilistic Buckling Analysis of SYLDA with Cut-outs

The load-shortening curve of the shell without any imperfections is shown along with the probability density function $\mathbf{f_{\Lambda_s}(\Lambda_s)}$ of $\Lambda_s(X)$ combining all sources of input imperfections.





Post-buckling shape of nominally perfect SYLDA with cut-outs at δ=2.50 mm.

The value of maximum load corresponding to **λ=0.87** is equal to **283 kN**.

Probabilistic Buckling Analysis of ISS

The reliability buckling factor λ of ISS is assessed for a reliability level equal to **99%** in the different analysis cases.

The NASA knockdown factor[8] is determined on the assumption that ISS can be considered a laminated composite shell of seven plies.

Analysis case	Reliability Buckling factor, λ	Load [kN]
Analysis of nominally perfect shell	1	545
Analysis with geometric imperfections	0.84	456
Analysis with variability in longitudinal Young's modulus	0.96	523
Analysis with ply misoreintation	0.99	540
Analysis with boundary imperfections	0.92	501
Analysis with combined imperfections	0.79	431
NASA knockdown factor[8]	0.64	349

ISS is more sensitive to the geometric imperfections than to other types of input imperfections.

8. V. I. Weingarten, P. Seide, and J. P. Peterson, NASA SP-8007 - Buckling of Thin-Walled Circular Cylinders, National Aeronautics and Space Administration, Washington, DC, USA (1968).

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Conclusions

A probabilistic methodology for a first assessment of the structural reliability of cylindrical shells under compression load is developed in order to determine the **Reliability Buckling Factor λ**.

This factor measures the sensitivity of the shell to sources of input imperfections, that are probabilistically treated, but depends highly on:

- **Required Standard of Reliability R_e.**
- **Adopted Sample Size N.**
- **Modeling of Input Random Parameters**.

The discussed procedure entails the advantage to be **versatile**:

- It is applicable to the buckling analysis of laminated composite shells and sandwich composite shells.
- Types of input imperfections and ways of their introduction into the numerical model different from the ones here proposed can be used.