PFH PRIVATE UNIVERSITY of Applied Sciences

DESICOS – new robust **DES**ign guideline for **I**mperfection sensitive **CO**mposite launcher **S**tructures

The Ritz method for the analysis of imperfect laminated composite cylinders and cones

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Braunschweig, 25th of March, 2015

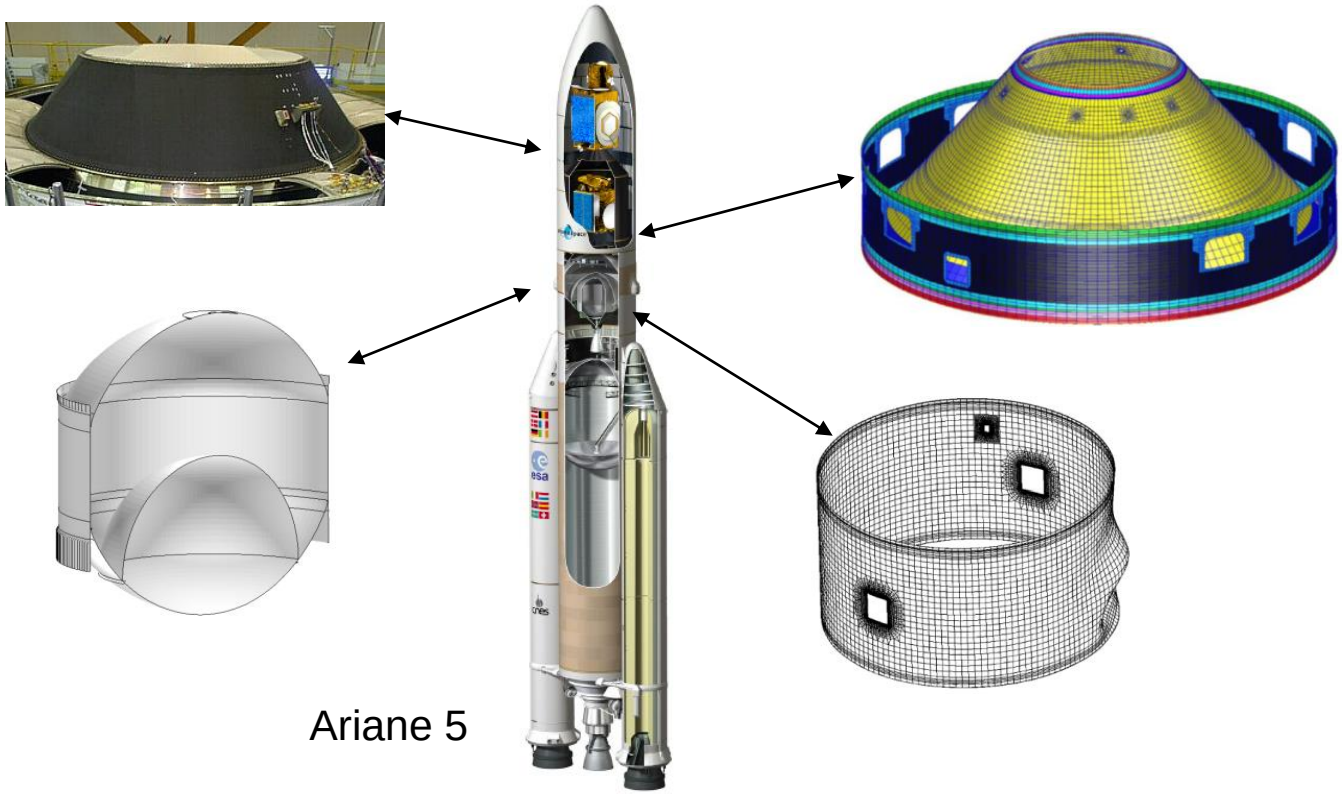
TU Clausthal

EMBRAER

SEVENTH FRAMEWORK PROGRAMME

Introduction

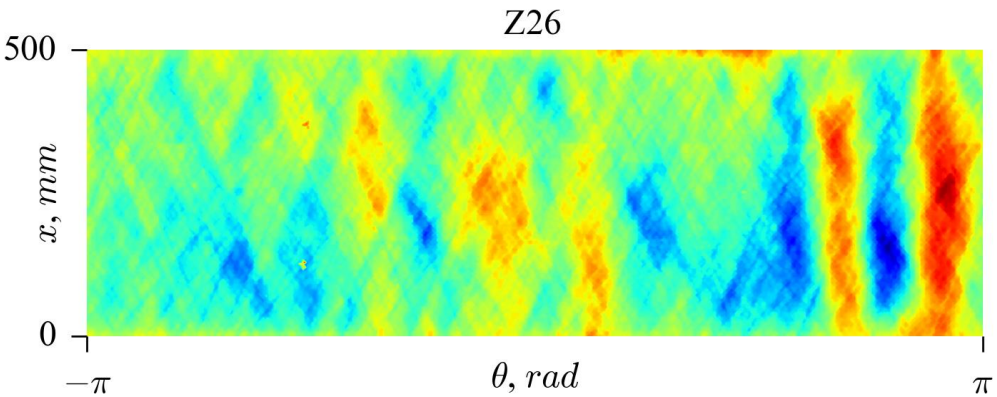
Unstiffened structures (Space):



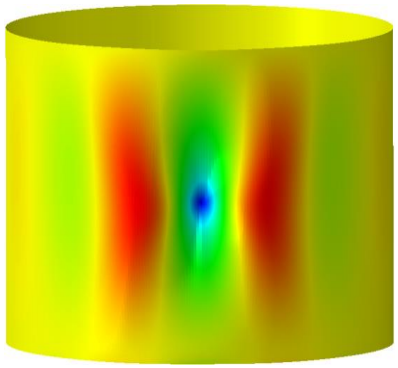
- ✓ **Sensitive to Geometric Imperfection**

Model Requirements

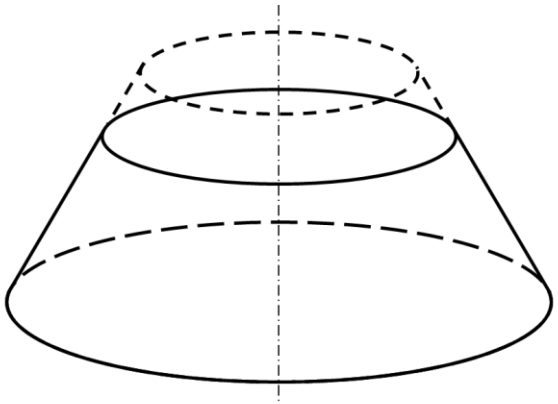
- ✓ **Geometric imperfections**
  - **Measured (courtesy of DLR):**



  - **Perturbation loads:**



- ✓ **Axial shortening**

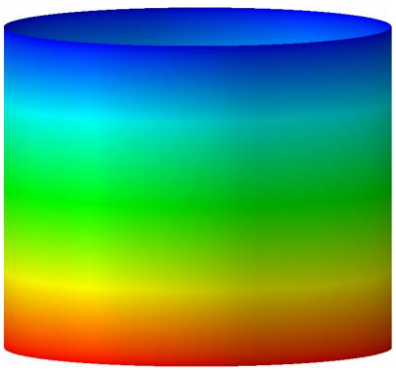


- ✓ **Elephant foot effect**

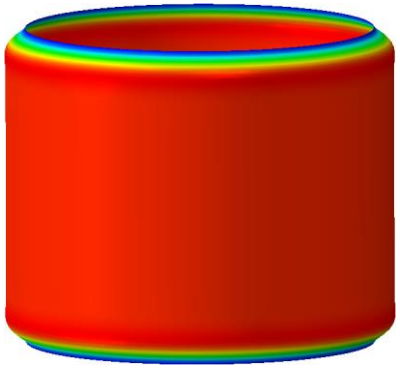
Ritz Method: Proposed Approximation Functions

$$\{u\} \approx \underbrace{[g_0]\{c_0\}} + \underbrace{[g_1]\{c_1\}} + \underbrace{[g_2]\{c_2\}}$$

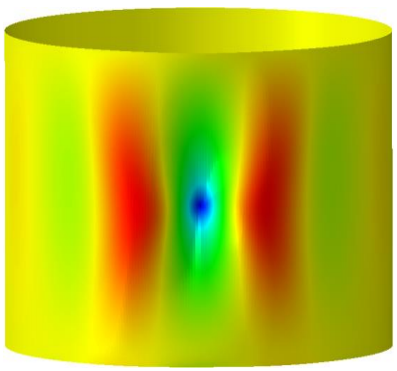
Axial Shortening



Axisymmetric

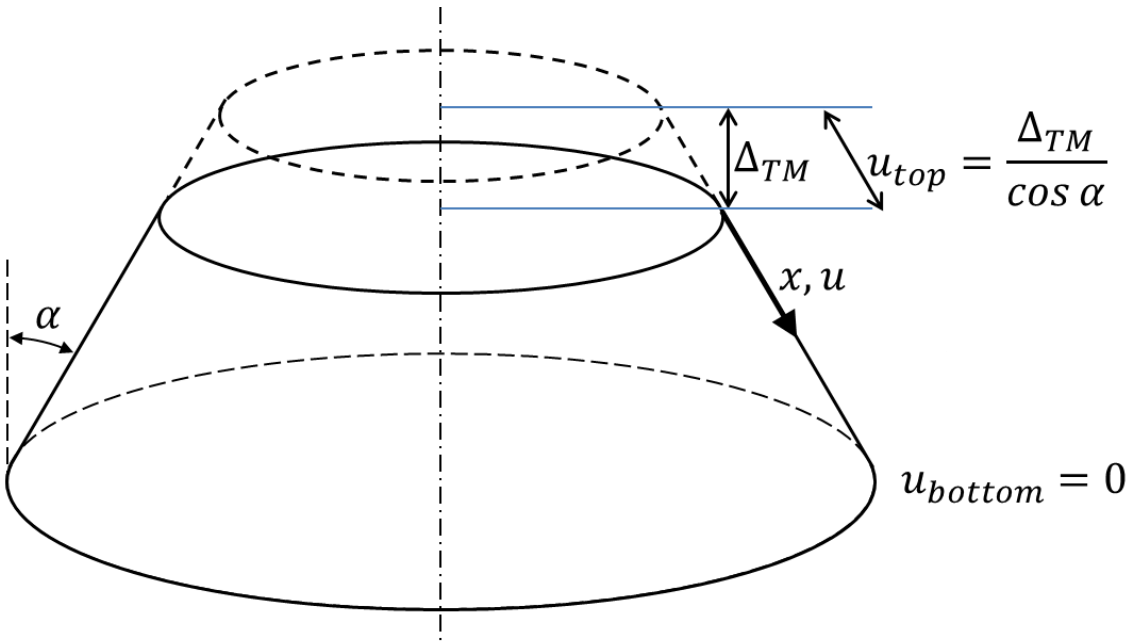


Two-dimensional field



Proposed Approximation Functions – Axial Shortening

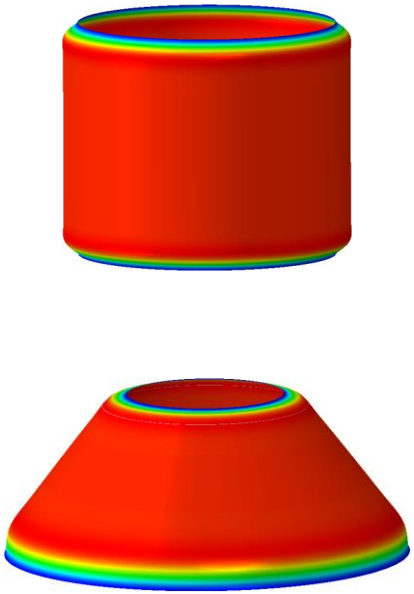
$$\{u\} \approx \underbrace{[g_0]\{c_0\}} + [g_1]\{c_1\} + [g_2]\{c_2\}$$



$$[g_0] = \begin{bmatrix} \frac{L - x}{L \cos\alpha} \\ 0 \\ 0 \end{bmatrix}$$

Proposed Approximation Functions – Axisymmetric

$$\{u\} \approx [g_0]\{c_0\} + \underbrace{[g_1]\{c_1\}} + [g_2]\{c_2\}$$



$$[g_1]_{CLPT} = \begin{bmatrix} & sin\left(i_1\pi\frac{x}{L}\right) & 0 & 0 & \\ \cdots & 0 & sin\left(i_1\pi\frac{x}{L}\right) & 0 & \cdots \\ & 0 & 0 & sin\left(i_1\pi\frac{x}{L}\right) & \end{bmatrix}$$

- **Not interested on the axisymmetric displacements u and v at the edges**

Proposed Approximation Functions – Two-dimensional

$$\{u\} \approx [g_0]\{c_0\} + [g_1]\{c_1\} + \underbrace{[g_2]\{c_2\}}$$

$$[g_2]_{CLPT} = \begin{bmatrix} \cdots & g^u_{ij_a} & g^u_{ij_b} & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & g^v_{ij_a} & g^v_{ij_b} & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & g^w_{ij_a} & g^w_{ij_b} & \cdots \end{bmatrix}$$

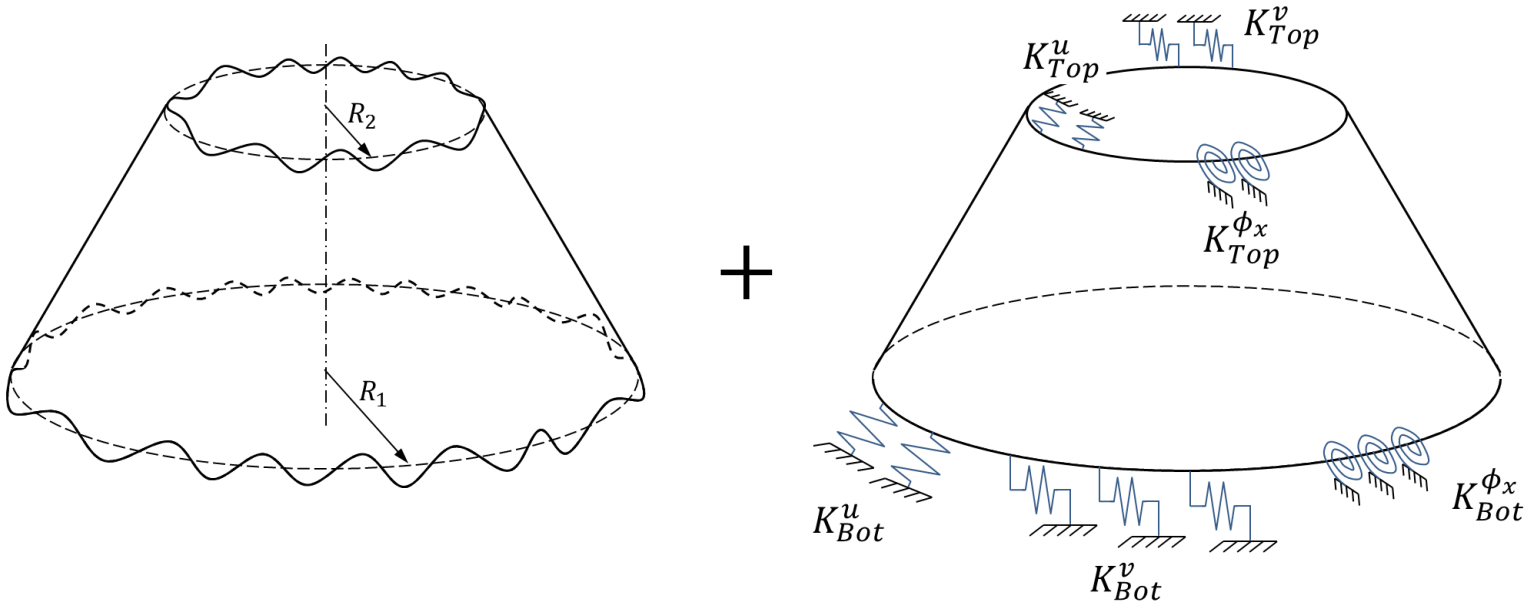
$$g^u_{ij_a} = cos\left(i_2\pi\frac{x}{L}\right)sin(j_2\theta) \qquad g^u_{ij_b} = cos\left(i_2\pi\frac{x}{L}\right)cos(j_2\theta)$$

$$g^v_{ij_a} = cos\left(i_2\pi\frac{x}{L}\right)sin(j_2\theta) \qquad g^v_{ij_b} = cos\left(i_2\pi\frac{x}{L}\right)cos(j_2\theta)$$

$$g^w_{ij_a} = sin\left(i_2\pi\frac{x}{L}\right)sin(j_2\theta) \qquad g^w_{ij_b} = sin\left(i_2\pi\frac{x}{L}\right)cos(j_2\theta)$$

- **How to simulate SS1 boundary conditions ($u = v = w = 0$) with these shape functions?**

Achieving Different Boundary Conditions



$$[K_0] + [K_e] = [K_{0_e}]$$

SS1:	$u = v = w = 0$	$K^u = K^v = \infty$
SS2:	$v = w = 0$	$K^v = \infty$
SS3:	$u = w = 0$	$K^u = \infty$
SS4:	$w = 0$	–

- **K^{ϕ_x} is used to switch between simply supported and clamped**
- **Infinity is implemented as 10^8, stable with double precision**

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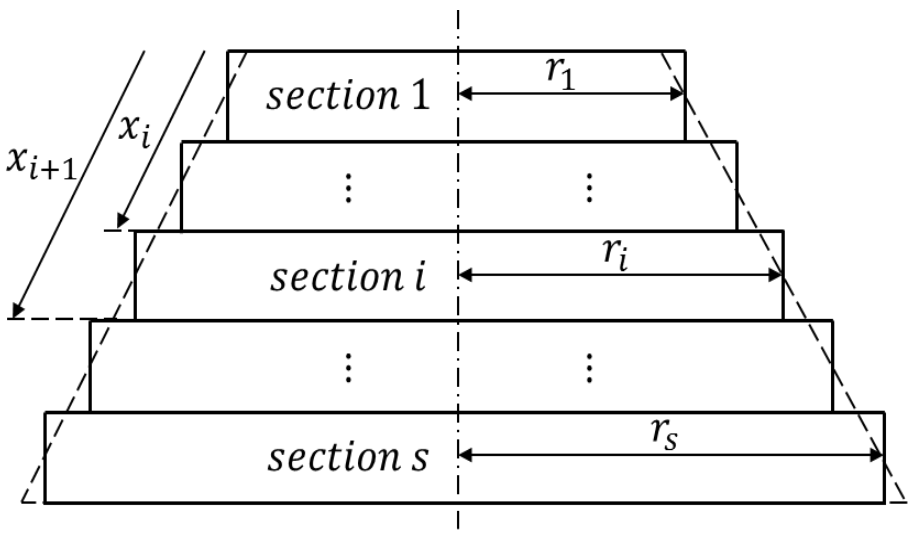
Implementation Aspects – Analytical Integration for Cones

$$\oint \int_0^L \frac{1}{r(x)} f(x,\theta,\alpha)\, dx\, d\theta = \oint \sum_{i=1}^{s} \int_{x_i}^{x_{i+1}} \frac{1}{r_i}\, f(x,\theta,\alpha)\, dx\, d\theta$$



- **Computational cost to compute $[\boldsymbol{K_0}]$ increases linearly with $\boldsymbol{s}$**
- **Using $\boldsymbol{s = 1}$ is already satisfactory**

Results

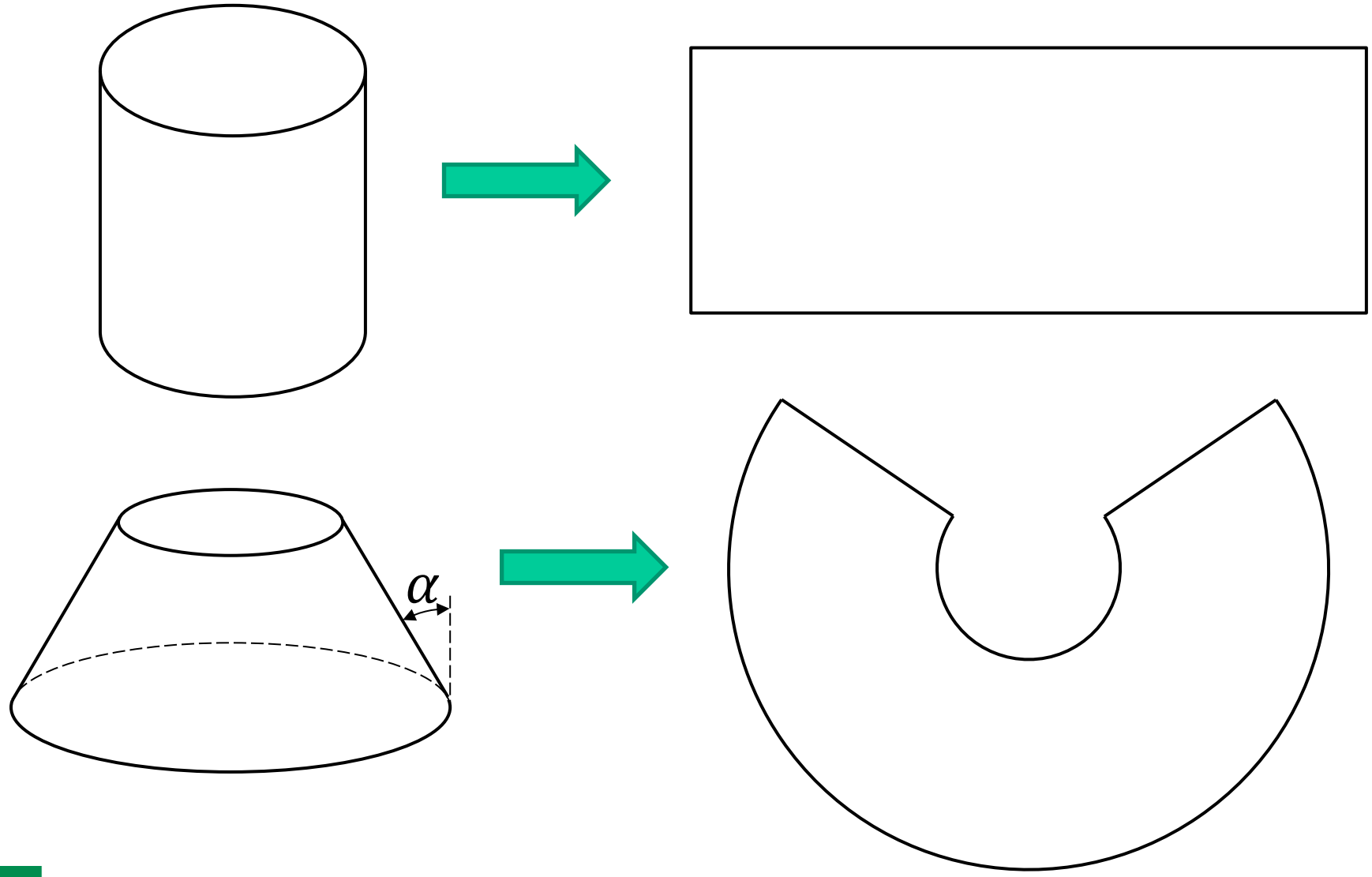
- Linear Buckling
- Linear Static
- Non-Linear Static

Structures Data

Cone / cylinder name	Reference	Material	R_1 (mm)	H (mm)	α (degrees)	Ply thickness (mm)	Stacking sequence Inwards – outwards
Z33	[24], [22]	Geier 2002	250	510	0	0.125	[0/0/19/-19/37/-37/45/-45/51/-51]
C02	None	Deg Cocomat	400	200	45	0.125	[30/-30/-60/60/$\overline{\mathbf{0}}$]sym

Material name	Reference	E_{11}	E_{22}	ν_{12}	G_{12}	G_{13}	G_{23}
Geier 2002	[22]	123.55	8.708	0.319	5.695	5.695	5.695
Deg Cocomat	[23]	142.50	8.700	0.28	5.100	5.100	5.100

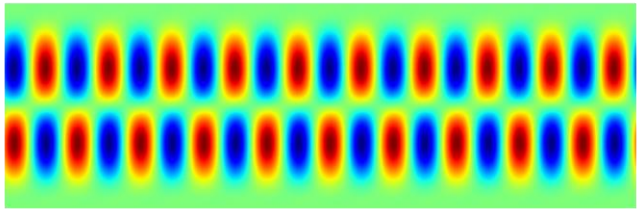
Visualization with Opened Shells



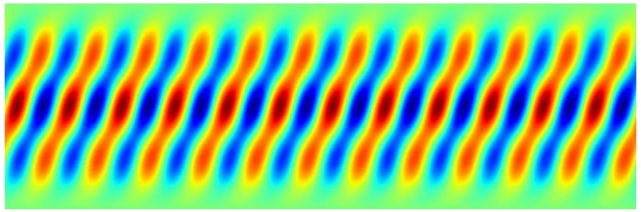
Linear Buckling Results (Cylinder Z33 – Convergence 1st Mode)

$m_1 = 120,\ m_2 = 5,\ n_2 = 5$

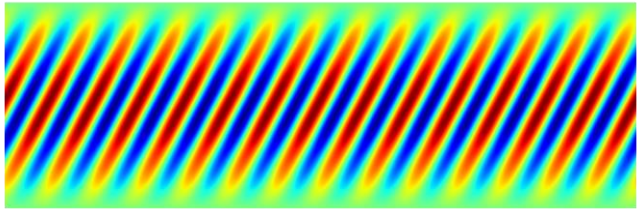
$m_1 = 120,\ m_2 = 10,\ n_2 = 10$



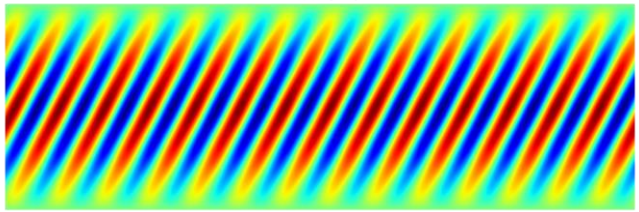
$m_1 = 120,\ m_2 = 12,\ n_2 = 12$

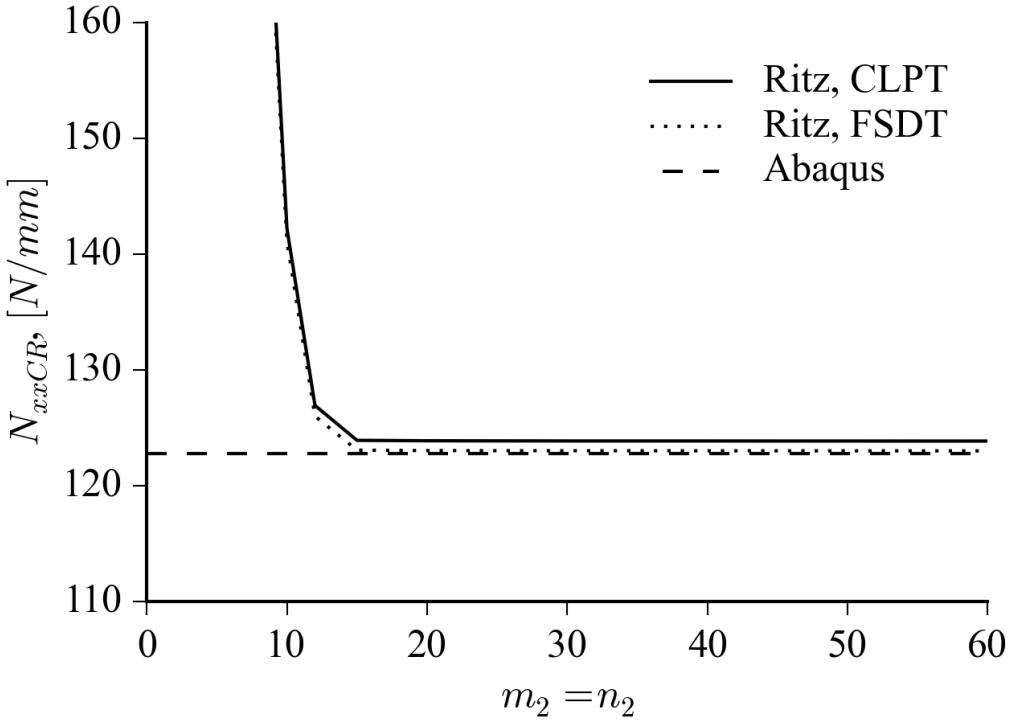


$m_1 = 120,\ m_2 = 20,\ n_2 = 20$



Abaqus, mode 1, $N_{xxCR} = 122.78,\ N/mm$





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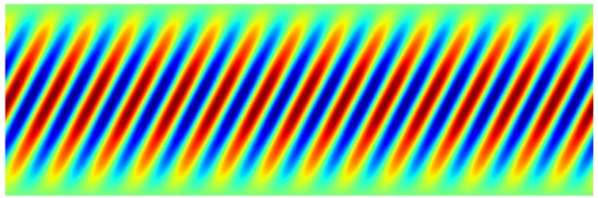
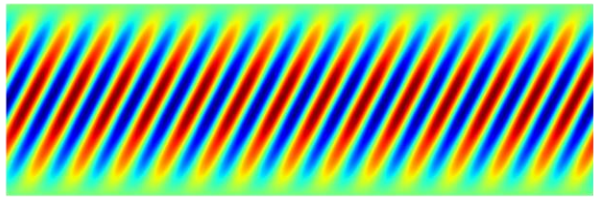
Linear Buckling Results (Cylinder Z33 – Verification with Abaqus)

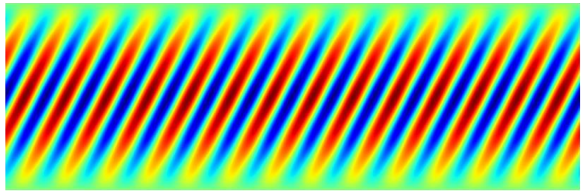
Ritz (CLPT), mode 1, $P_{CR}=194.55kN$



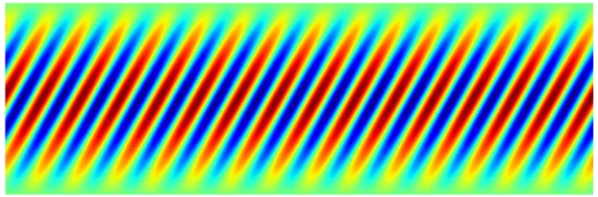
Ritz (FSDT), mode 1, $P_{CR}=193.22kN$



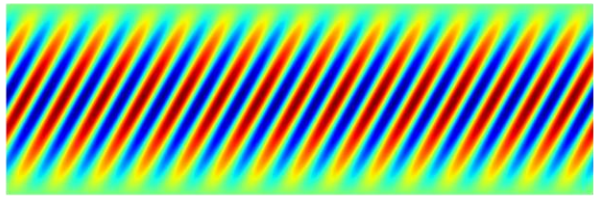
Abaqus, mode 1, $P_{CR}=192.86kN$



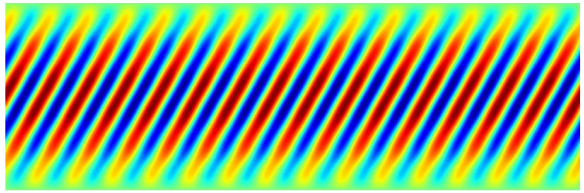
Ritz (CLPT), mode 5, $P_{CR}=197.22kN$



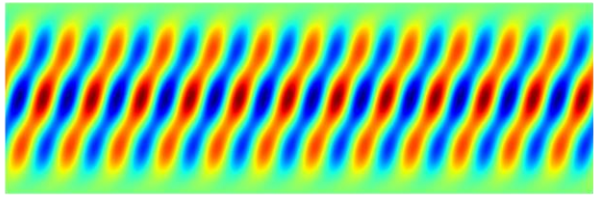
Ritz (FSDT), mode 5, $P_{CR}=195.87kN$



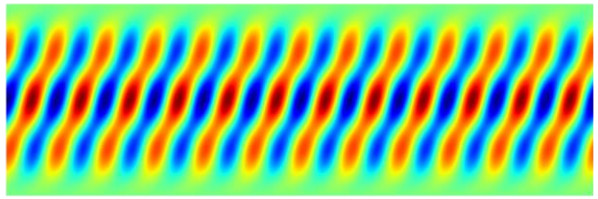
Abaqus, mode 5, $P_{CR}=195.75kN$



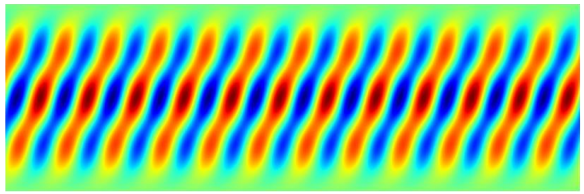
Ritz (CLPT), mode 7, $P_{CR}=199.09kN$



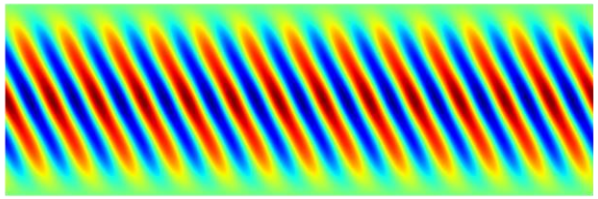
Ritz (FSDT), mode 7, $P_{CR}=197.63kN$



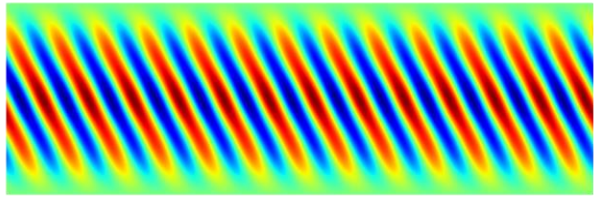
Abaqus, mode 7, $P_{CR}=196.63kN$



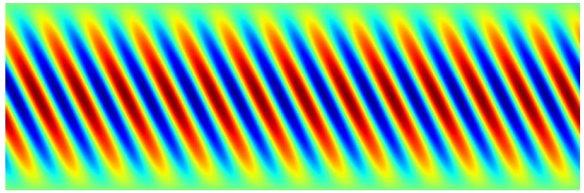
Ritz (CLPT), mode 9, $P_{CR}=200.52kN$



Ritz (FSDT), mode 9, $P_{CR}=199.08kN$



Abaqus, mode 9, $P_{CR}=198.10kN$

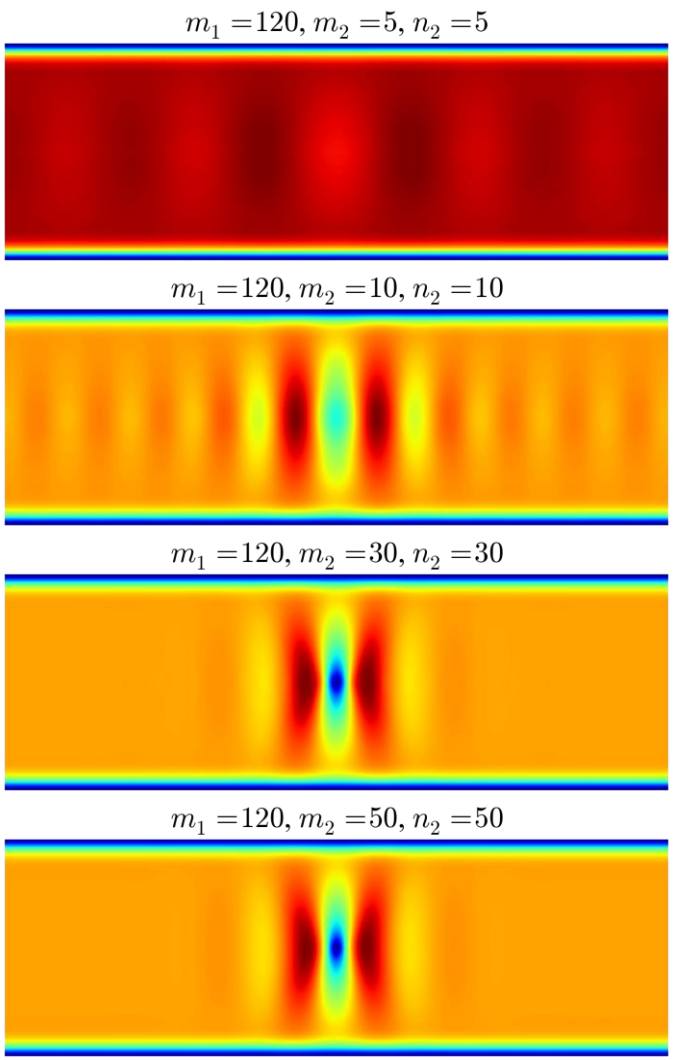


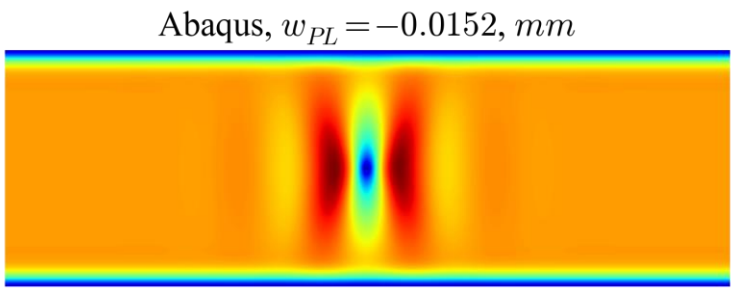
Linear Buckling Results (Cylinder Z33 – Computational Cost)

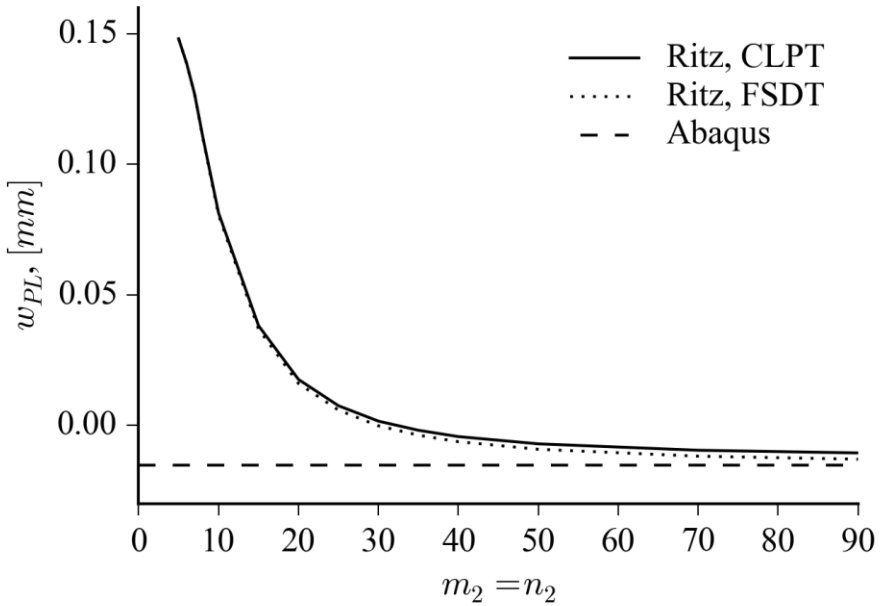
Elements (S4R) around θ	$F_{C_{crit}}$ (kN)	Elapsed Time (s)
120	214.147	12
160	203.497	20
200	198.800	37
240	196.318	43
280	194.842	66
320	193.868	134
420	192.391	142

	CLPT, Donnell		CLPT, Sanders	
m_2 and n_2	$F_{C_{crit}}$ (kN)	Elapsed Time (s)	$F_{C_{crit}}$ (kN)	Elapsed Time (s)
10	223.402	0.22	221.720	0.24
15	194.633	0.32	193.569	0.31
20	194.576	0.52	193.511	0.54
25	194.561	0.76	193.496	0.84
30	194.550	1.19	193.486	1.22
35	194.546	1.74	193.481	1.93
40	194.542	2.79	193.477	2.95
50	194.538	5.29	193.473	5.83

Linear Static Results (Cylinder Z33 – Convergence and Verification)







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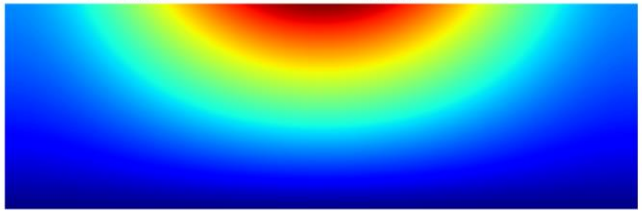
Linear Static Results (Cylinder Z33 – Computational Cost)

Elements (S4R) around θ	u_{TM} (mm)	w_{PL} (mm)	Elapsed time (s)
120	0.0449	-0.0259	2
160	0.0449	-0.0267	5
200	0.0449	-0.0270	7
240	0.0449	-0.0272	10
280	0.0449	-0.0274	13
320	0.0449	-0.0275	19
420	0.0449	-0.0276	32

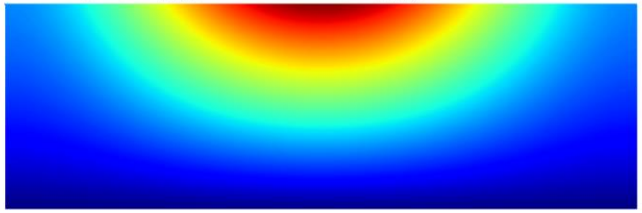
m_2 and n_2	u_{TM} (mm)	w_{PL} (mm)	Elapsed time (s)
20	0.0449	-0.0194	0.08
40	0.0449	-0.0249	0.40
50	0.0449	-0.0256	0.78
60	0.0449	-0.0260	1.32
70	0.0449	-0.0262	2.21
80	0.0449	-0.0263	3.36
90	0.0449	-0.0264	5.05
100	0.0449	-0.0265	6.94
110	0.0449	-0.0266	10.22
120	0.0449	-0.0266	13.68

Non-Linear Static Analysis with Asymmetric Loads (Z33)

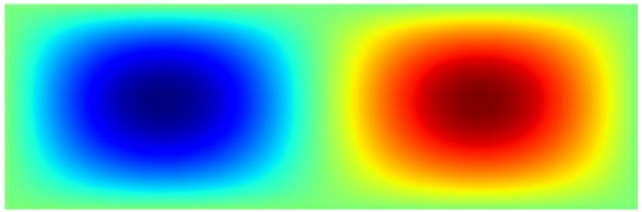
Ritz, $u_{min} = -0.00000$, $u_{max} = 0.71492mm$



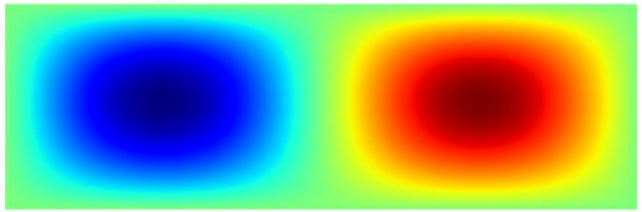
Abaqus, $u_{min} = 0.00000$, $u_{max} = 0.71980mm$



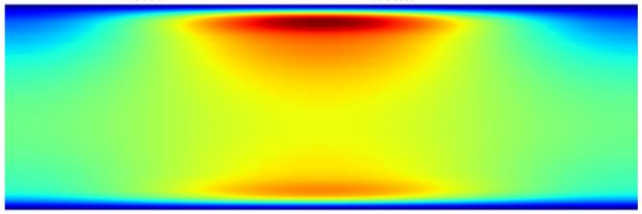
Ritz, $v_{min} = -0.06874$, $v_{max} = 0.06874mm$



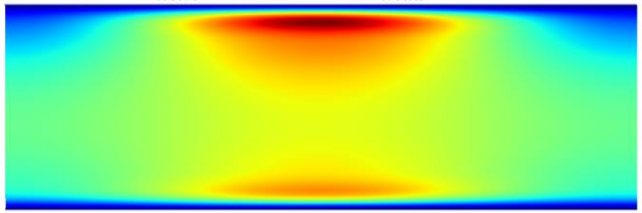
Abaqus, $v_{min} = -0.06879$, $v_{max} = 0.06878mm$



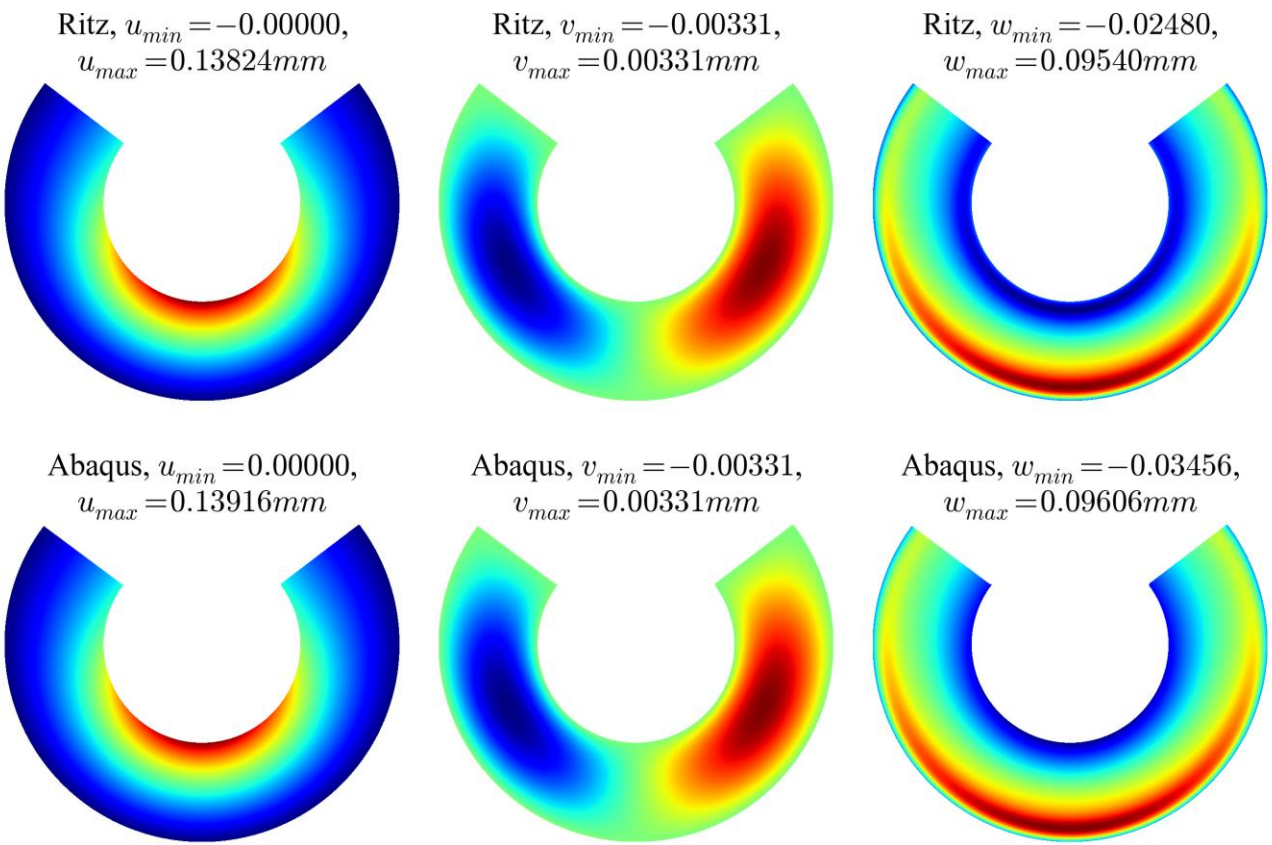
Ritz, $w_{min} = -0.00182$, $w_{max} = 0.28482mm$



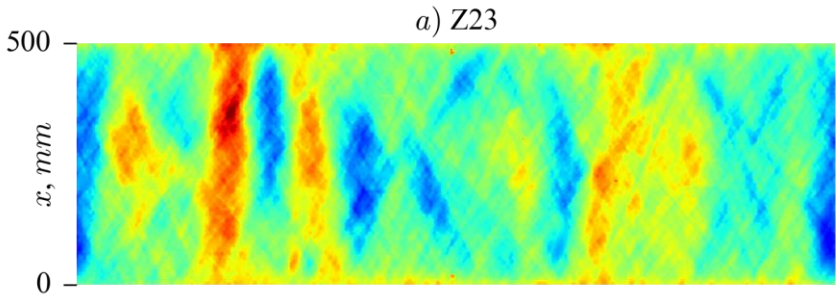
Abaqus, $w_{min} = 0.00000$, $w_{max} = 0.28761mm$



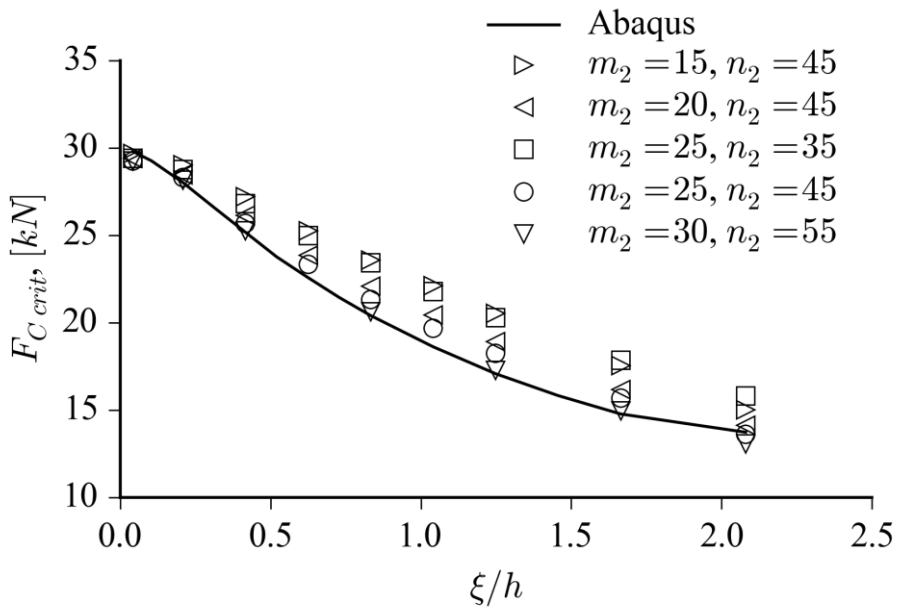
Non-Linear Static Analysis with Asymmetric Loads (C02)



Non-Linear Analysis of an Imperfect Cylinder



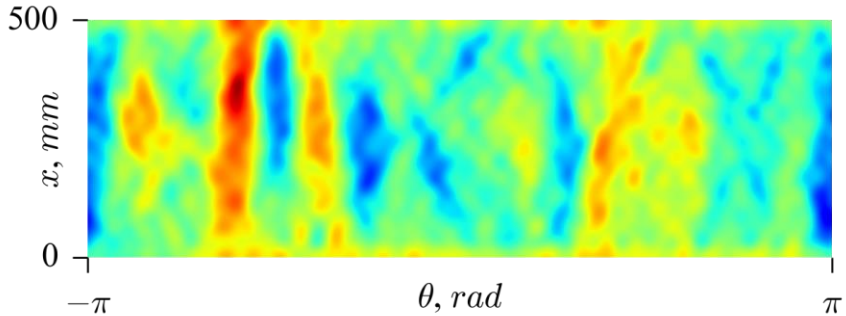
(measured by DLR)



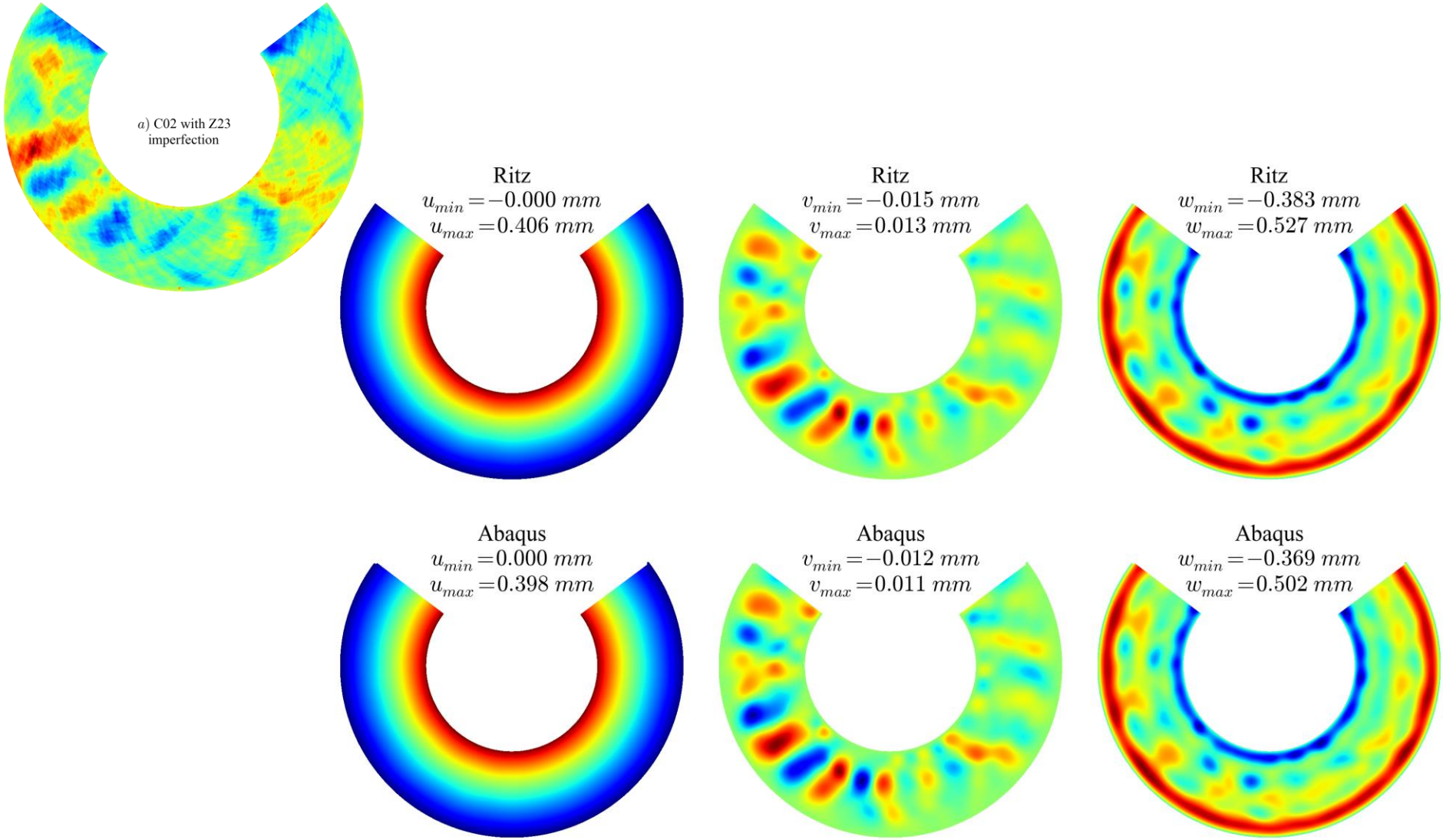
Arbocz's half-cosine function (1968):

$$w_0(x,\theta) = \sum_{j=0}^{n_0}\sum_{i=0}^{m_0} cos\left(\frac{i\pi x}{L}\right)\left(A_{ij}\, cos(j\theta) + B_{ij}\, sin(j\theta)\right)$$

with: $m_0 = 20$ and $n_0 = 30$



Non-Linear Analysis of an Imperfect Cone



Non-Linear Buckling Analysis with Perturbation Loads (Cylinder Z33)

	Abaqus (420 elements around circumference, S4R)		**Ritz** (CLPT, Donnell, $m_1 = 120$, $m_2 = 25$, $n_2 = 45$)		
Perturbation Load (N)	Fc_{crit} (kN)	Elapsed time (s)	Fc_{crit} (kN)	Elapsed time (s)	**Error**
1	192.490	992	193.279	6914	**0.41%**
5	190.430	1048	192.399	8199	**1.03%**
20	167.285	708	169.094	7336	**1.08%**
30	148.998	777	150.173	5836	**0.79%**
45	123.070	773	124.970	6672	**1.54%**
60	101.392	724	102.634	4920	**1.23%**
70	88.975	612	90.059	4533	**1.22%**

- **Average Error = 1.04 %**
- **Ritz method ~10X slower**

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Conclusions

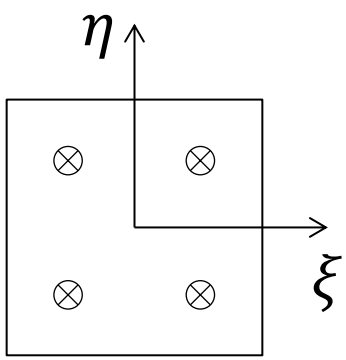
1) **The semi-analytical models achieved their purpose:**
   - ✓ **Inclusion of geometric imperfections**
   - ✓ **Torsion, pressure, axial compression**
   - ✓ **General axial load distributions**
   - ✓ **Different boundary conditions**

2) **Current implementation compared to Abaqus:**
   a) **~100 X faster for linear buckling analysis**
   b) **~5 X faster for linear static analysis**
   c) **~10 to 100 X slower for non-linear static analysis**

Reason for the Slow Numerical Integration in the Ritz Method

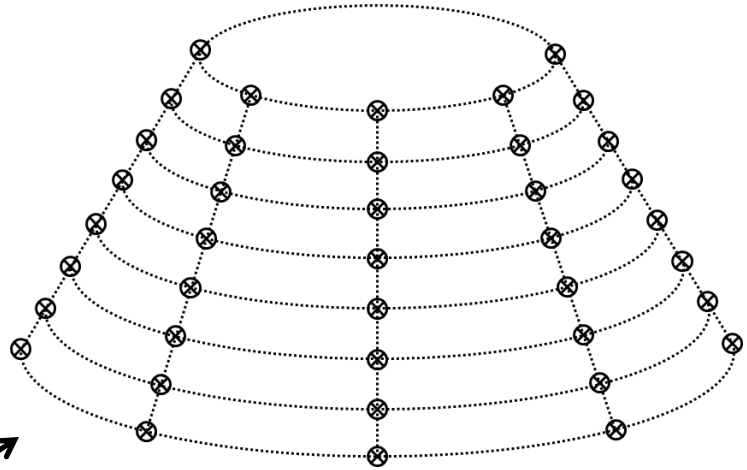
1) Numerical integration in finite elements

$$[K_{global}] = \left[\quad [K_{elem}] \quad \right]$$



2) Numerical integration in the Ritz method

$$[K] = \left[\left[\quad K_{structure} \quad \right]\right]$$



Suggested Topics for Research

1) **Efficient numerical integration for the (non-linear) Ritz method**

   → **Example: application of quadrature rules**

2) **Non-homogeneous laminate properties:**

   → **Work with the governing equations (Differential Quadrature Method)**

Publications

Saullo G. P. Castro, Christian Mittelstedt, Francisco A. C. Monteiro, Mariano A. Arbelo, Richard Degenhardt. “A semi-analytical approach for the linear and non-linear buckling analysis of imperfect unstiffened laminated composite cylinders and cones under axial, torsion and pressure loads”. **Thin-Walled Structures**, 90, 61-73, 2015.

Saullo G. P. Castro, Christian Mittelstedt, Francisco A. C. Monteiro, Mariano A. Arbelo, Gerhard Ziegmann, Richard Degenhardt. “Linear buckling predictions of unstiffened laminated composite cylinders and cones under various loading and boundary conditions using semi-analytical models”. **Composite Structures**, 118, 303-315, 2014.

Saullo G. P. Castro , Christian Mittelstedt, Francisco A. C. Monteiro, Richard Degenhardt, Gerhard Ziegmann. “Evaluation of non-linear buckling loads of geometrically imperfect composite cylinders and cones with the Ritz method”. **Composite Structures**, 122, 284-289, 2015.

Thank you!

- questions?

- All the development available at:
  - http://compmech.github.io/compmech/

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