Deterministic and stochastic buckling analysis for imperfection sensitive stiffened cylinders

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- design concept with knock-down factors
- deterministic analysis
- stochastic analysis
- combined knock-down factor
- conclusions
The concept of knock-down factors – introduction.

- Formula: \( F_{\text{design}} = F_{\text{perfect}} \times k \)
  - \( k < 1 \) knock-down factor

Graph showing the relationship between load (\( f \)) and displacement (\( u \)) for perfect and imperfect structures.
concept of knock-down factors – introduction

- standard design approach based on NASA SP-8007 (1968)
- provides lower-bound curves from experimental data
concept of knock-down factors – introduction

- experimental testing & numerical prediction improved
- SP-8007 seems to be too conservative

Example: CFRP cylinder
- Total length = 540 mm
- Free length = 500 mm
- Ply orientation = +24,-24,+41,-41
- Radius = 250 mm
- Thickness = 0.5 mm
- $R/t = 500$
- $F_{\text{perfect}} = 32 \text{ kN}$

CFRP – carbon fibre reinforced polymer
**less conservative** design approach proposed, based on numerical simulation results

**old:** \[ F_{\text{design}} = F_{\text{perfect}} \times k_{\text{nasa}} \]

**new:** \[ F_{\text{design}} = F_{\text{perfect}} \times k_1 \times k_2 \]

\( k_1 \) considers **geometric imperfection** using deterministic methods
\( k_2 \) considers **other imperfections** using stochastic methods
The **new design concept** was tested exemplarily with two **stiffened** test cylinders.

<table>
<thead>
<tr>
<th>id</th>
<th>material $E, \mu$</th>
<th>cylinder radius</th>
<th>cylinder height</th>
<th>skin thickness</th>
<th>stiffener thickness</th>
<th>stiffener height</th>
<th>stiffener number</th>
<th>NASA SP8007 knock-down factor</th>
<th>Test ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>70000, 0.34</td>
<td>400</td>
<td>1000</td>
<td>0.8</td>
<td>0.8</td>
<td>5.2</td>
<td>90</td>
<td>0.4616</td>
<td>?</td>
</tr>
<tr>
<td>B</td>
<td>70000, 0.34</td>
<td>400</td>
<td>1000</td>
<td>0.55</td>
<td>0.55</td>
<td>5.2</td>
<td>126</td>
<td>0.4387</td>
<td>YES</td>
</tr>
</tbody>
</table>

Two different **numerical models** were used:

- stringer shell model
- smeared shell model
buckling analysis – **stringer shell model**

- explicitly modeled shell stringers
- 174960 S4R shell elements (Abaqus)
- S4R: reduced integration to avoid locking
- hourglass modes exist

**discretization**

- axial directions: 216 elements
- between two stringers: 6 elements
- stiffener height: 3 elements
buckling analysis – **smeared shell model**

- no modeled shell stringers
- 25100 S4R shell elements (Abaqus)
- less elements (factor 7)
- consideration of measured geometric imperfections of unstiffened cylinders

\[
K = \begin{bmatrix}
73747.59668 & 21528.72 & 0 & 31283.4956 & 0 & 0 \\
21528.72 & 63319.7648 & 0 & 0 & 0 & 0 \\
0 & 0 & 20895.5224 & 0 & 0 & 0 \\
31283.4956 & 0 & 0 & 120724.922 & 1148.1984 & 0 \\
0 & 0 & 0 & 1148.1984 & 3377.05412 & 0 \\
0 & 0 & 0 & 0 & 0 & 1321.9469 \\
\end{bmatrix}
\]
### Buckling Analysis - Comparison Model A

- Number of stiffeners: 90
- Thickness skin/stiffener: 0.8 mm

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Linear Buckling Load $F_{\text{perfect}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stringer model</td>
<td>205.92 kN</td>
</tr>
<tr>
<td>(174960 elements)</td>
<td></td>
</tr>
<tr>
<td>Smeared model</td>
<td>203.27 kN (rel. dev 1.29%)</td>
</tr>
<tr>
<td>(25100 elements)</td>
<td></td>
</tr>
</tbody>
</table>

First buckling mode:
- **Stringer model**
- **Smeared model**

** TU Delft **

**Challenge the future**
buckling analysis – **comparison model B**

- number of stiffeners: 126
- thickness skin/stiffener: 0.55 mm

<table>
<thead>
<tr>
<th>model type</th>
<th>linear buckling load</th>
<th>$F_{\text{perfect}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>stringer model</strong></td>
<td></td>
<td>103.09 kN</td>
</tr>
<tr>
<td>(174960 elements )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>smeared model</strong></td>
<td></td>
<td>103.76 kN (rel. dev 0.65%)</td>
</tr>
<tr>
<td>(25100 elements )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **first buckling mode**
  - **stringer model**
  - **smeared model**
analysis design – **deterministic study**

\[ F_{\text{design}} = F_{\text{perfect}} \times k_1 \times k_2 \]

- \( k_1 \) considers **geometric imperfection** using deterministic methods
- \( k_2 \) considers **other imperfections** using stochastic methods

**methods used to model geometric imperfections**

- single perturbation load approach (SPLA) applied to the **stringer model**
- modeling of measured imperfections (Z15, Z17, Z20) applied to the **smeared model**
knock-down curves – deterministic study

**single perturbation load approach** applied to stiffener model
- SPL on stiffener
- SPL in skin

![Graph](image-url)

- SPL on stiffener: global buckling
- SPL on stiffener: first buckling
- SPL on skin: global buckling
- SPL on skin: first buckling
imperfection approach applied to smeared model – cylinder \( A \)

with averaged knock-down factors from results of three measurements \( Z_{15}, Z_{17}, Z_{20} \)
imperfection approach applied to smeared model – cylinder B

with averaged knock-down factors from results of three measurements Z15, Z17, Z20
knock-down factors – deterministic study

<table>
<thead>
<tr>
<th>method</th>
<th>cylinder A</th>
<th>cylinder B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 bar</td>
<td>0 bar</td>
</tr>
<tr>
<td></td>
<td>0.2 bar</td>
<td>0.2 bar</td>
</tr>
<tr>
<td>SPLA</td>
<td>0.620</td>
<td>0.640</td>
</tr>
<tr>
<td></td>
<td>0.800</td>
<td>0.828</td>
</tr>
<tr>
<td>meas. geometric imperfections</td>
<td>0.621 (rel dev. 0.29%)</td>
<td>0.638 (rel dev. 0.31%)</td>
</tr>
<tr>
<td></td>
<td>0.785 (rel dev. 1.87%)</td>
<td>0.804 (rel dev. 2.89%)</td>
</tr>
</tbody>
</table>

- here: sufficient correspondence
- $k_1$ used from single perturbation load approach
\[ F_{\text{design}} = F_{\text{perfect}} \times k_1 \times k_2 \]

- \(k_1\) considers **geometric imperfection** using deterministic methods
- \(k_2\) considers **other imperfections** using stochastic methods

**cases considered**

1. geometric imperfection **not included**
   applied to the smeared model to obtain \(k_2\)

2. geometric imperfection (Z15, Z17, Z20) **included**
   applied to the smeared model for comparison with new KDF
Monte Carlo simulation based on ABAQUS

- buckling considered as probabilistic phenomenon due to distribution of input parameters

- scatter of input parameters
  - material
  - thickness
  - geometric imperfection
  - load imperfection

- nonlinear buckling analyses

- analysis results
  - results provide distribution of buckling loads
  - lower bound defined with 99% confidence level determines KDF

Matlab

ABAQUS

Python & Matlab
assumed normal distribution of input parameters (material, thickness skin & stiffener, applied compressive load) with

- a coefficient of variation (CV) = 5% (measure of dispersion)
  - $\sigma$: standard variation
  - mean $\mu$ := initial design / measured value
- number of samples used: 5000
- examples: modulus of elasticity, applied load

\[
CV = \frac{\sigma}{\mu}
\]
input parameter distribution – stochastic study

used checks for normal distribution of the input parameter
mean $\mu = \text{initial design / measured value}$

1. histogram

2. cumulative distribution function (CDF)

3. Lilliefors test: data accept the normal hypothesis with a 99% confidence level
- CV (coef. of variation) of load imperfection was varied: 3% 5% 10%

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</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>0 bar</td>
</tr>
<tr>
<td>geometric imperfections not included</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CV=3%</td>
<td>0.86</td>
<td>0.85</td>
</tr>
<tr>
<td>CV=5%</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>CV=10%</td>
<td>0.81</td>
<td>0.79</td>
</tr>
<tr>
<td>stochastic with geometric imperfections included</td>
<td>Z15</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>Z17</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Z20</td>
<td>0.68</td>
</tr>
</tbody>
</table>
**combined knock-down factors – design values**

\[ F_{\text{design}} = F_{\text{perfect}} \times k_1 \times k_2 \]

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<th>cylinder B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 bar</td>
<td>0.2 bar</td>
</tr>
<tr>
<td>( k = k_1 \times k_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>geometric imperfections ( k_1 )</td>
<td>CV=3%</td>
<td>0.53</td>
</tr>
<tr>
<td>geometric imperfections ( k_2 )</td>
<td>CV=5%</td>
<td>0.52</td>
</tr>
<tr>
<td>geometric imperfections ( k_2 )</td>
<td>CV=10%</td>
<td>0.50</td>
</tr>
<tr>
<td>other imperfections ( k_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>stochastic with geometric imperfections <strong>included</strong></td>
<td>Z15</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>Z17</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Z20</td>
<td>0.68</td>
</tr>
</tbody>
</table>
combined knock-down factors – design values

cylinder A – 0 bar

- $k_{(\text{NASA})} = 0.46$
- $k_{(3\%)} = 0.53$ (16%)
- $k_{(5\%)} = 0.52$ (14%)
- $k_{(10\%)} = 0.50$ (9%)


cylinder B – 0 bar

- $k_{(\text{NASA})} = 0.44$
- $k_{(3\%)} = 0.54$ (24%)
- $k_{(5\%)} = 0.53$ (21%)
- $k_{(10\%)} = 0.50$ (15%)

NASA SP-8007
combined knock-down factors – design values

cylinder B – 0.2 bar

- $k_{\text{Seide}} = 0.648$
- $k_{(3\%)} = 0.74 \ (14\%)$
- $k_{(5\%)} = 0.72 \ (11\%)$
- $k_{(10\%)} = 0.69 \ (7\%)$
summary / conclusions

- **buckling performance** of two stiffened cylinders was analysed
- **smeared model** used
  - considers measured geometric imperfections
  - reduces computational complexity in stochastic MC-based analysis
- **two knock-down factors** derived
  - $k_1$ deterministic analysis $\rightarrow$ geometric imperfections
  - $k_2$ stochastic analysis $\rightarrow$ other imperfections (load, material,...)
- combined approach is
  - robust and less conservative compared to NASA SP8007
  - more conservative than a pure stochastic approach
DESICOS

New Robust DESign Guideline for Imperfection Sensitive COMposite Launcher Structures

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RWTH Aachen Germany
TECHNION Israel

CRC-ACS – Coop. Research Australia Centre for Adv. Composite Structures
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